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Evaluating the Performance of Incremental Sampling

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Table of Contents

Introduction....2
Example Case Study....2
Description of the Simulated Sites....7
RDX 10 x10 m Decision Unit 1....8
RDX 10 x 10 m Decision Unit 2 & HMX 10 x 10 m Decision Unit....8
NG 10 x 10 Decision Unit....8
2,4 DNT 10 x 10 m Decision Unit....8
Simulated Sampling Pattern Descriptions....10
Simulation Results....15
NG 10 x 10 Decision Unit....17
RDX 10 x 10 Decision Unit....20
RDX 10 x 10 Decision Unit 2....23
2-4 DNT 10 x 10 Decision Unit....26
HMX 10 x 10 Decision Unit....29
Conclusions....32
References....35
Appendix A – Simulated Site Development Documentation....36
Appendix B – Simulation Results on other ITRC Developed Sites....47
Averaged Increment Results....48
Summed Increment Results....52

Introduction

During the past two years the US Army Corps of Engineers (USACE) has drafted interim guidance on incremental sampling (US Army Corps of Engineers, 2009) and the Interstate Technology & Regulatory Council (ITRC) began an incremental sampling methodology team to provide guidance to the states on the uses of incremental sampling (http://www.itrcweb.org/teampublic_ISM.asp). John Hathaway has been a contributor to the ITRC working group and is one of the primary participants on the incremental sampling (IS) simulations and statistics sub group. John Hathaway also works with Brent Pulsipher on the Visual Sample Plan (VSP) software development team (vsp.pnl.gov). VSP is developed by Pacific Northwest National Laboratory (PNNL) for statistical sampling and analysis applications with environmental data. Mr. Hathaway and Mr. Pulsipher developed Incremental statistical sampling tools in VSP (currently identified as Multiple Increment Sampling) that are different from the currently applied methods proposed by the USACE interim guidance. This report was developed to help with the ITRC incremental sampling group and to provide a starting point for aligning VSP and the USACE interim guidance in our application of incremental sampling.

We will focus on simulation studies we performed to evaluate the performance of some of the statistical aspects associated with incremental sampling under a variety of simulated site conditions. There are many other important features that are being addressed as a part of the ITRC incremental sampling group (i.e. applicable constituents, laboratory procedures, sample gathering protocols, etc.). Our simulation studies primarily focus on the effects of the incremental sampling process as applied to identifying an appropriate estimate for the mean concentration of a decision unit.

Example Case Study

Before we present the full simulation study and the associated results, we provide a case study from the simulation to describe our perspective of how an analysis would be performed with incremental sampling data.

Suppose through the use of a detailed conceptual site model (CSM) a clearly identified firing point is present. The site team has blocked a 10 x 10 m decision unit (DU) on which they would like to use incremental sampling to identify the average concentration (mg/kg) of RDX in the topsoil. Through team meetings, it is agreed that 4 incremental samples (IS) will be analyzed. Each incremental sample contains 49 (7x7 pattern) increments or discrete grabs from the DU.

For this work we developed four ways that the 4 IS could be gathered from within the DU. Figure 1 shows a graphical representation of each of these four sampling patterns within a 10 x 10 m DU. The upper left and right sampling designs are both built on a systematic grid design. In this design a random location is selected for the first increment in each IS and the other 48 increments are gathered systematically based on the first increment location. The lower two patterns are built around a random grid design. For a random grid design the region for each IS is separated into a 7x7 grid, similar to the systematic designs, and a random location is selected within each of the 49 grid nodes to build each IS sample. The two images on the left side of Figure 1 represent the standard IS procedures in current use while the two images on the right identify a grouped IS method we propose in this report.

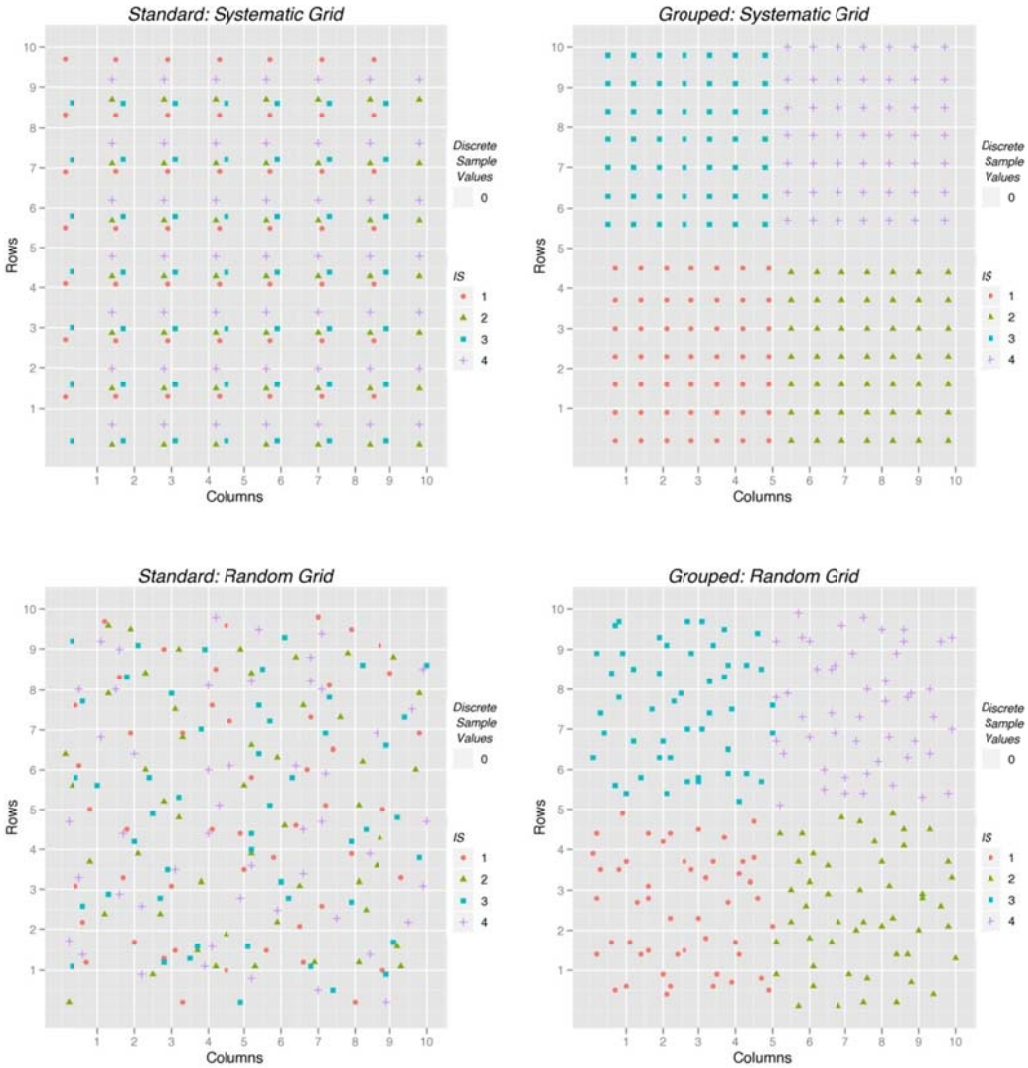

Figure 1: Representation of four possible methods of gathering 4 IS of 49 increments.

The information shown in Table 1 represent RDX concentrations in mg/kg for each of the 4 IS and each sampling pattern that could be observed. The associated means, standard deviations, and two different upper confidence levels (UCL) are shown for each sampling pattern. The appropriate method to calculate a UCL will be discussed later, but these UCL values were calculated at 95% confidence using a *t*-distribution with 3 degrees of freedom (# IS – 1). The UCL SE line shows the upper confidence level using the standard error, where the standard error is $SD / \sqrt{\# IS - 1}$.

Table 2 shows the results for two discrete data samples (DS) of 9 locations. The first column identifies a random sample of 9 points and the second column identifies a systematic grid sample of 9 points over the DU. We have avoided the calculation of a UCL on the discrete data sets, but we do show the calculated standard deviations (SD) and standard errors (SE) where the SE are $SD / \sqrt{\# DS - 1}$.

We suppose that in general application only one of the columns from Table 1 or Table 2 would be obtained for the decision unit and all inference would be made from that respective column of data. However, the remaining evaluation of this case study will compare and contrast all 6 data sets. These

results are only one realization of each method from the DU and should not be used to compare and contrast the methods. The results from the simulation studies in the following sections provide a much clearer framework for comparison.

With respect to the IS designs, it is interesting to note that for this particular realization, the means are all generally close to each other regardless of the sampling pattern, but the standard deviations differ considerably. This difference will be discussed in more detail in the simulation summary section.

The discrete sampling designs do not have as consistent performance due to the limited number of discrete samples (9). While only one example, it is clear that a set of 9 discrete samples from this DU will generally provide a wide range of mean and standard deviation estimates.

Table 1. Summary of the 4 IS values obtained for each IS sampling pattern.

	Standard Random Grid	Grouped Random Grid	Standard Systematic Grid	Grouped Systematic Grid
IS 1	54.54	19.42	46.03	40.07
IS 2	64.5	119.13	35.81	166.43
IS 3	65.24	1.69	139.86	1.92
IS 4	55.13	109.18	60.06	68.9
Mean	59.85	62.36	70.44	69.33
SD	5.81	60.39	47.33	70.31
St. Error (SE)	3.35	34.87	27.33	40.59
UCL SE	67.74	144.41	134.75	164.86

Table 2. Summary of the 2 discrete data sets for random and systematic sampling.

	Random Discrete Sample	Systematic Discrete Sample
DS 1	7.836	5.931
DS 2	0.1713	154.821
DS 3	5.483	7.335
DS 4	3.001	28.560
DS 5	1.348	3.729
DS 6	1.453	5.084
DS 7	0.275	2.997
DS 8	0.136	1.256
DS 9	0.011	443.776
Mean	2.190	72.610
SD	2.828	52.680
St. Error	1.000	18.625

Another observation between the standard IS, grouped IS, and discrete samples lies in the reliability and quantity of spatial information. The discrete data provide the most spatial information within the DU, but the reliability of each discrete point representing the average constituent level around it is more questionable. The standard IS data will provide the most reliable estimates for the constituent

contamination within the local region they are estimating. However, that region is the entire DU and as such no spatial information within the DU is obtained. The grouped IS method will provide reliable results for each of the regions from which they are sampled and some spatial information is maintained.

Except for the random discrete samples, all of the respective data sets from this single realization indicate that the mean concentration of RDX is around 70 mg/kg with UCL values as high as 234 mg/kg. The systematic discrete data show some high levels (one above 400 mg/kg) and elucidate that the distribution of RDX on the site is very heterogeneous. For both of the grouped IS designs, IS 3 identifies a region of the site that has much lower concentrations than the other four regions. This additional spatial information can be useful if remediation of the DU is required.

While brief, this case study gives a snapshot of the simulation results from sampling performed on a 10 x 10 m DU and potential conclusions that could be made. Figure 2 depicts a spatial map of the values for each increment that could be obtained from the simulated DU. This simulated site is based on the general patterns identified from data obtained during studies performed at Fort Polk, Al as a part of demonstrations of Incremental sampling (Jenkins, Thiboutot, Ampleman, & Hewitt, 2005). Figure 3 shows the same DU separated into 3 x 3 areas with each area having its own color scale to show the extreme range of values within the DU. These two figures identify that the DU had a very high elevated RDX region in the bottom middle and moderately high levels towards the middle on the left side of the DU. Most of the right side of the DU has lower levels of RDX (multiple orders of magnitude lower).

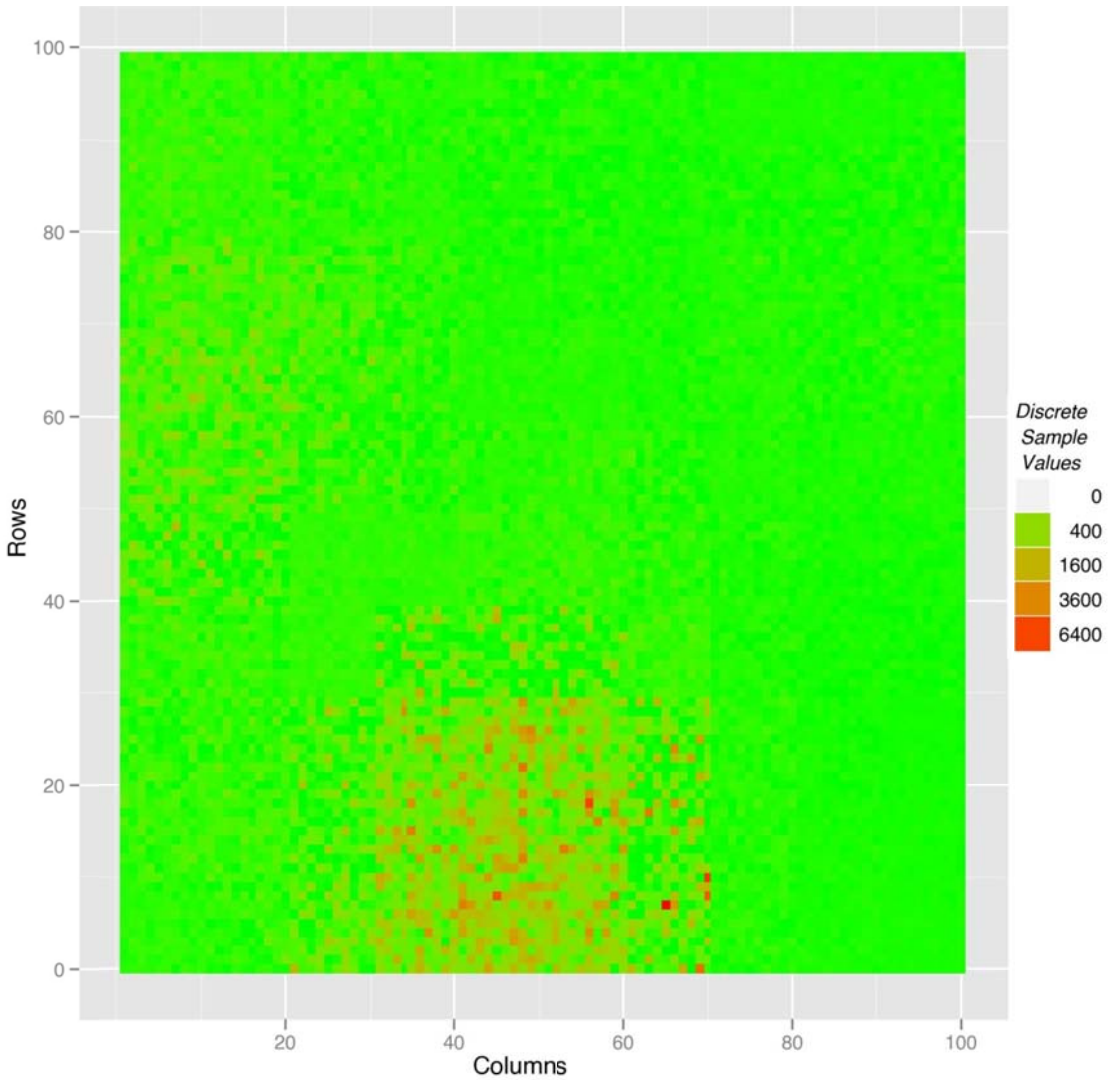

Figure 2. Spatial map of the simulated RDX values for each increment that could be sampled.

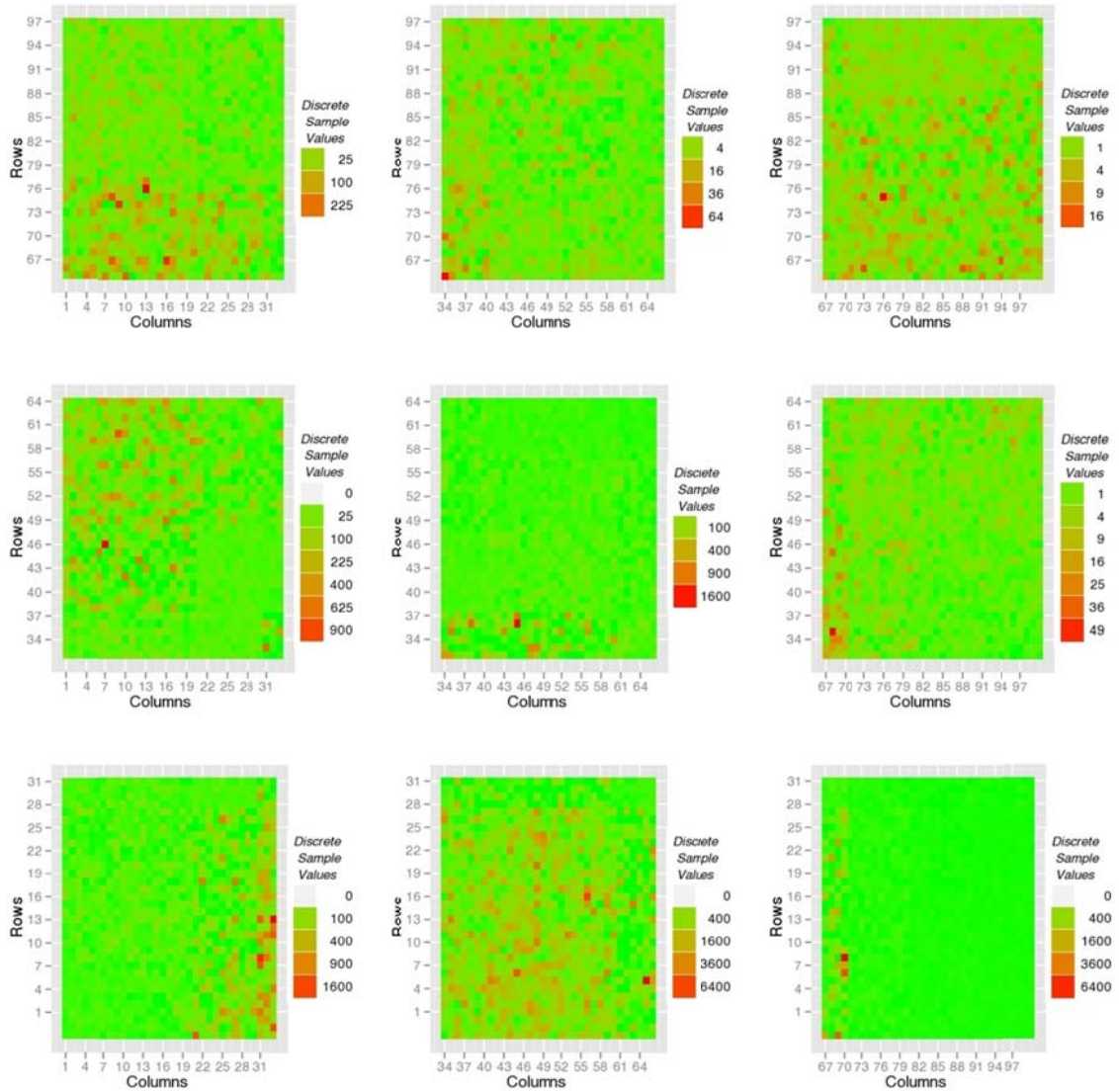

Figure 3. Separated (3x3) spatial map of the simulated RDX values for each increment that could be sampled. Each sub-region has its own color scale to provide a clearer picture of the scale over the site.

While none of the 6 sampling methods identified values as large as 6,400 mg/kg all but the random discrete sample were relatively close to the true mean of the simulated DU at 71.36 mg/kg. Once again, we should highlight that the 9 discrete samples will provide highly variable results for mean concentration estimates as is shown by the low mean estimate from the 9 discrete systematic samples.

The mean concentration of 71.36 mg/kg is the true mean for the DU, but there are large regions of this DU with spatial averages that are at least an order of magnitude higher than the mean.

Description of the Simulated Sites

We developed 5 simulated sites to evaluate the different sampling patterns through their statistical properties. Figures 2 and 3 represent one of the simulated sites for a 10x10 DU with concentration values for RDX. This site has the most diverse spatial structure and very high spatial heterogeneity.

Figure 4 shows the other 4 simulated sites we developed for this research. These sites still have extreme heterogeneity and are captioned with the respective munitions constituent values shown. Figure 5 shows each of these sites in two-dimensional histograms with a square-root transformed count axis to improve the visualization of the tail values. With a standard count axis shown these distributions would look even more extreme. Their respective means are marked with a dotted green vertical line. Each of these sites are based on the patterns and concentration values observed from data gathered as a part of ESTCP project led by T. Jenkins and A. Hewitt (Hewitt, Jenkins, Ramsey, & Bjella, 2005; Jenkins, Hewitt, Ranney, & Ramsey, 2004; Jenkins, Thiboutot, Ampleman, & Hewitt, 2005). For a more detailed presentation of how simulated sites were developed see appendix A.

RDX 10 x10 m Decision Unit 1

This DU was used as the example in the case study above. The values are RDX concentration values in mg/kg. This DU has the most extreme heterogeneity and is the heaviest right tailed distribution as shown in the far left histogram in Figure 5. Figures 1 and 2 show how the DU has one area with extremely high RDX concentration values (bottom middle) and a second area with high RDX concentration values (middle right side) while the rest of the DU has orders of magnitude lower concentration values. The mean concentration for this DU is 71.37 mg/kg of RDX.

RDX 10 x 10 m Decision Unit 2 & HMX 10 x 10 m Decision Unit

The spatial RDX and HMX concentrations (mg/kg) maps shown in the lower left and upper right of Figure 4 depict the second RDX decision unit and the HMX decision unit. These DUs have some spatial patterns, but they are much more dispersed (less heterogeneous) and the distribution of values is not as heavily right tailed (see second and third histograms in Figure 5). The maximum RDX concentration value for an increment is much lower than that for the RDX Decision Unit 1 (at 36 mg/kg for a single increment) and the maximum HMX value is 2.26 mg/kg. The mean concentration for the RDX DU 2 and HMX DU are 0.776 and 0.132 mg/kg respectively.

NG 10 x 10 Decision Unit

The NG decision unit is another site with concentration values that are heavily skewed (see second histogram from the right in Figure 5). This DU is the least skewed as can be seen by comparing histograms in Figure 5. The spatial concentration map shown in the lower right of Figure 4 displays that this DU has some spatial structure, but the concentration values have a much smoother pattern within the DU. The mean concentration for this DU is 2.82 mg/kg of NG.

2,4 DNT 10 x 10 m Decision Unit

The 2,4 DNT decision unit has many of the same patterns of the previously described DUs. The concentration values from this DU display a heavy right tail and the spatial map shown in the upper left of Figure 4 shows some spatial patterns to the concentration values. The mean concentration for this DU is 0.694 mg/kg of 2,4 DNT.

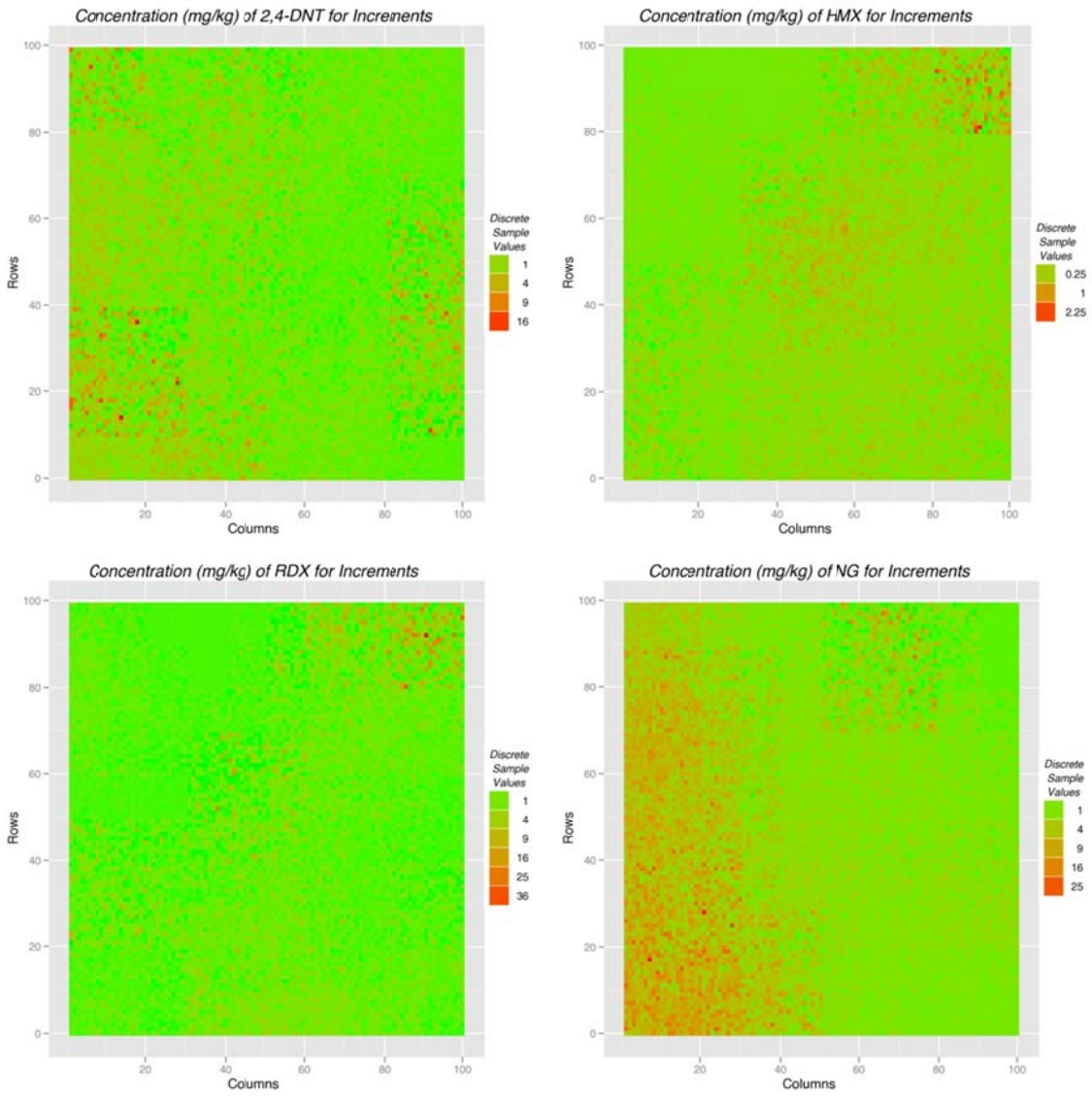

Figure 4. Four additional simulated decision units from which sampling simulations were performed. The range of concentration values and dispersion within the DU gets its basis from actual increment data gathered in previous studies of Incremental sampling (See appendix).

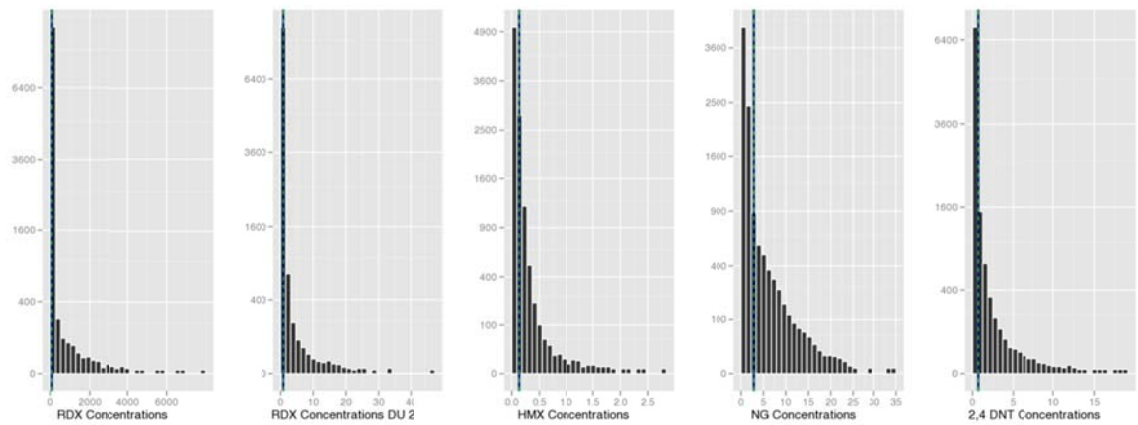

Figure 5. Histograms (Squareroot transformed y-axis) of the concentration values for each of the 5 simulated sites. The dashed green line identifies the mean concentration for each site. From left to right the means are 71.37, 0.776,0.132, 2.82, 0.694 mg/kg respectively.

Simulated Sampling Pattern Descriptions

Figures 6-9 show the 64 different IS patterns that were evaluated and summarized in the following simulation results section. For all four figures, each row of plots represent a different number of IS gathered from the DU (2, 3, 4, 5) and each column of plots identifies a different number of increments per IS (16, 30, 49, 100). Figures 6 and 7 show the standard IS procedure with replicate IS over the entire DU for systematic and random grid sampling respectively. Figures 8 and 9 represent the grouped IS methods for systematic and random grid sampling respectively. These Figures show the general structure for each of the evaluated patterns, but only represent an example of one random selection for each pattern.

Figure 10 shows the 9, 16, 30, and 100 random and systematic discrete sampling types that were evaluated. Once again, these examples show the general structure for each of the evaluated sampling types and only represent one random selection for each pattern.

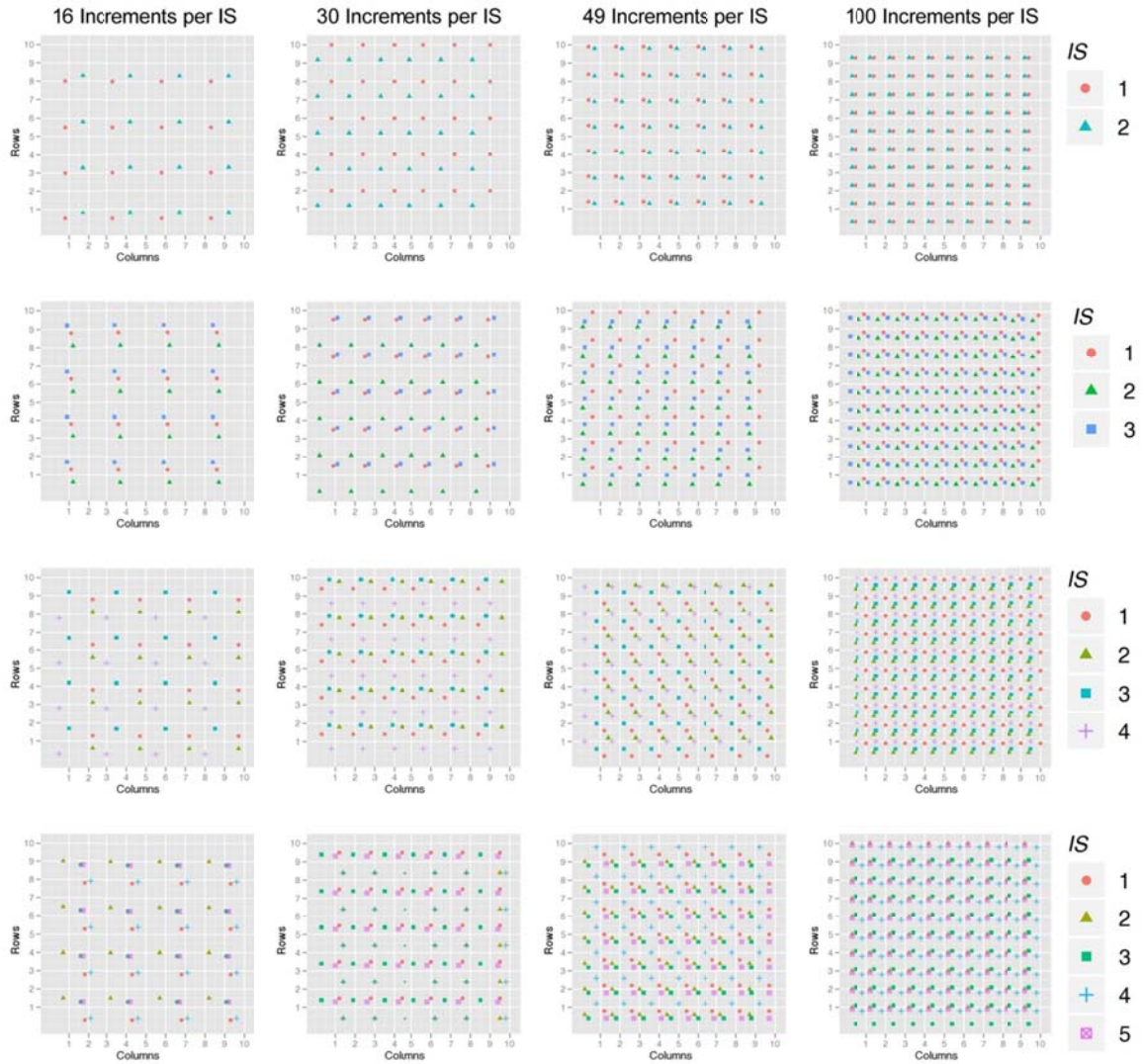

Figure 6. Standard incremental sampling using a systematic grid sampling approach. Each column represent a differing number of increment per IS and each row depicts the differing number of IS that were gathered.

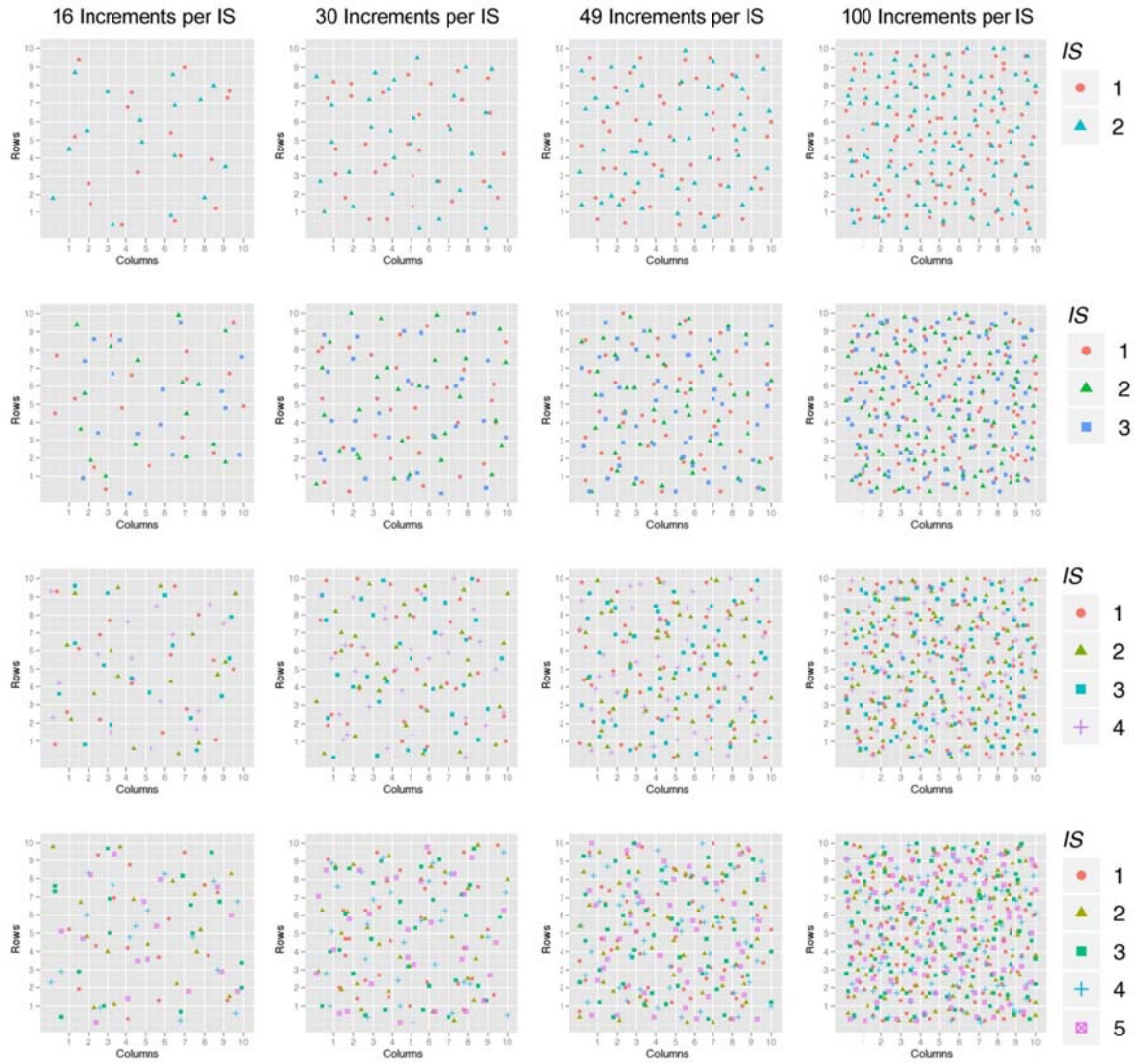

Figure 7. Standard incremental sampling using a random grid sampling approach. Each column represent a differing number of increment per IS and each row depicts the differing number of IS that were gathered.

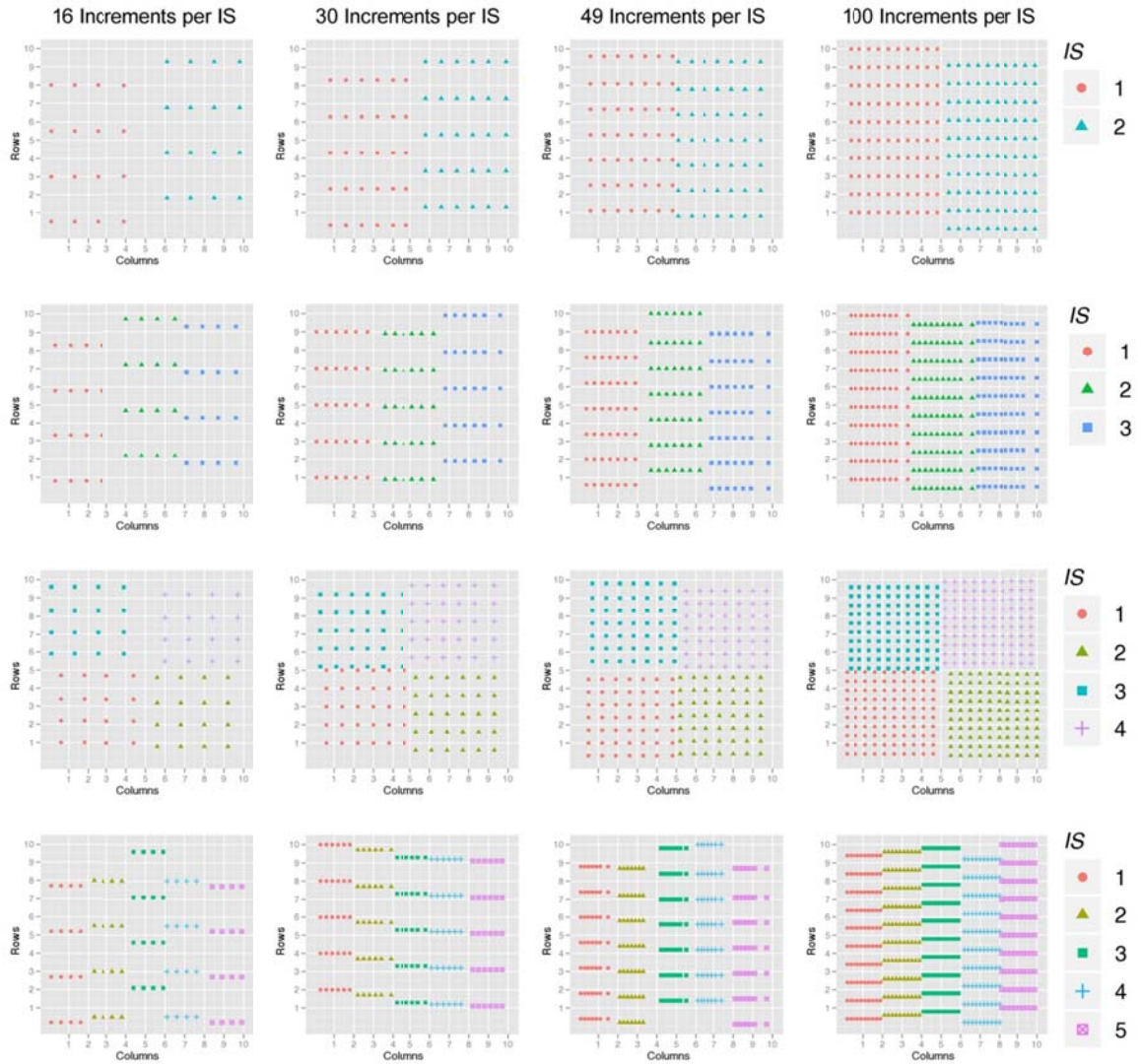

Figure 8. Grouped incremental sampling using a systematic grid sampling approach. Each column represent a differing number of increment per IS and each row depicts the differing number of IS that were gathered.

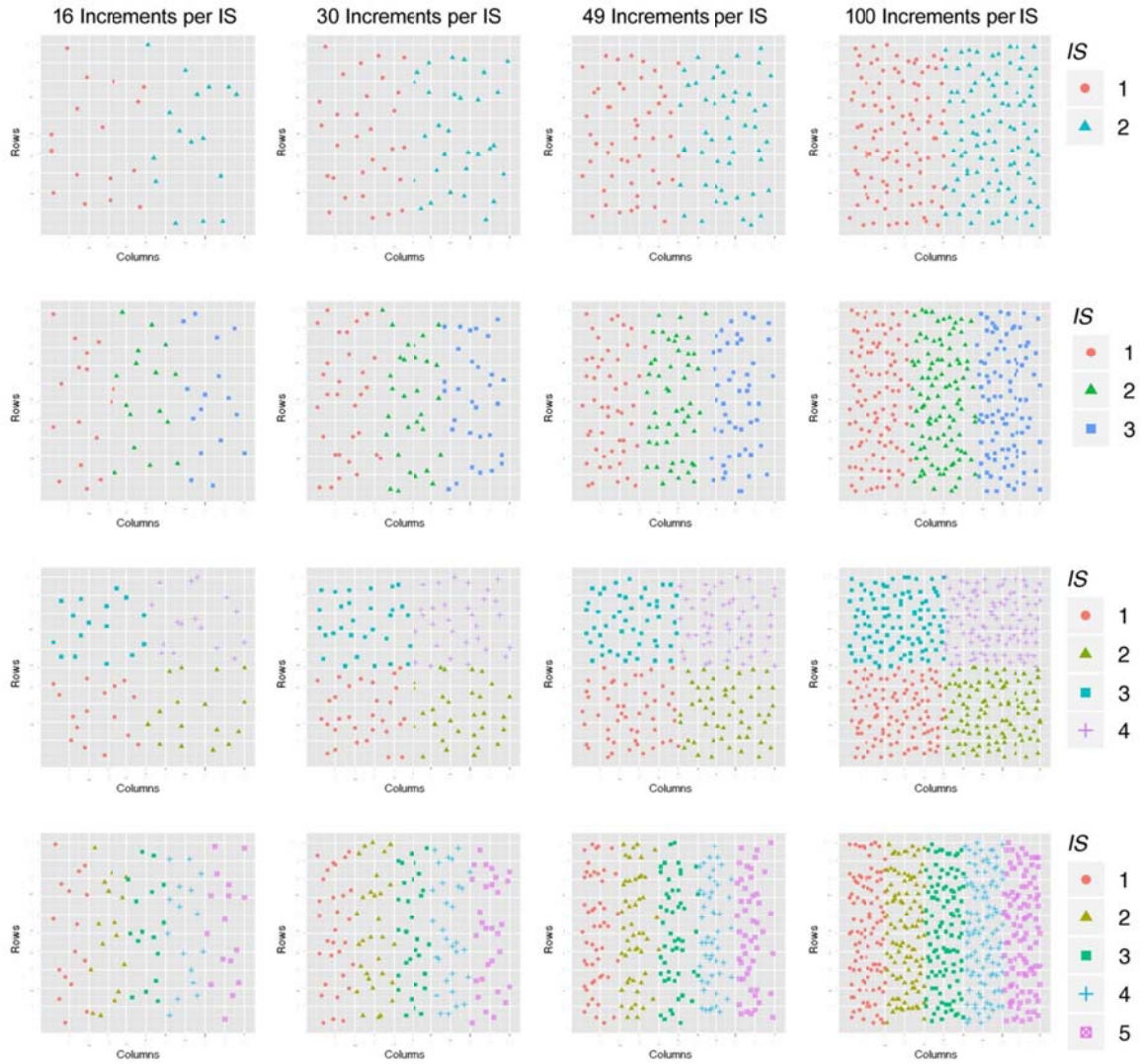

Figure 9. Grouped incremental sampling using a random grid sampling approach. Each column represent a differing number of increment per IS and each row depicts the differing number of IS that were gathered.

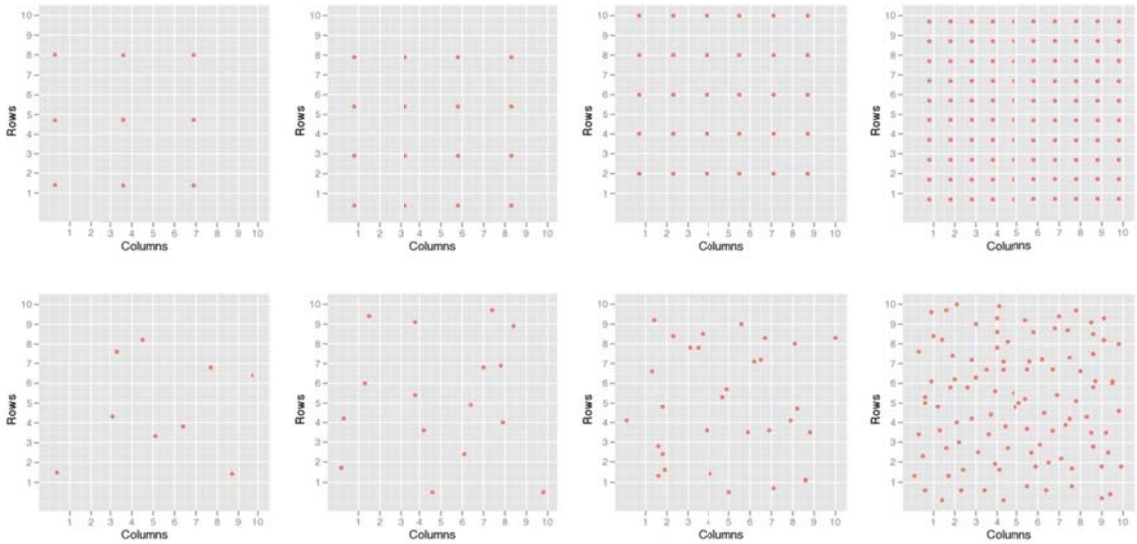

Figure 10. Discrete sampling using a systematic grid (top row) and random grid (bottom row) sampling approaches. Each column represent a differing number of increments or discrete samples. From left to right 9, 16, 30, & 100 samples per evaluation.

Simulation Results

The validity of confidence interval construction methodologies is commonly inspected through determining the given methodology's coverage. Coverage in this aspect does not include any 2-dimensional spatial connotation; rather, it refers to the probability that the methodology captures the parameter. If the methodology is sound, then the capture probability will be equal to the specified confidence level. Simulation is a common methodology for inspecting coverage. For UCL simulation calculations, coverage is a 1-dimensional comparison of the resulting UCL values over many iterations in a simulation to the known true mean used in the simulation process. Table 3 shows a few of the 2,000 iterations from the UCL calculations based on using the mean and standard error calculated from 9 systematic grid discrete samples (see upper left plot in Figure 10) on the RDX 2 site (see lower left plot in Figure 4). The values from the UCL column are then compared to the true mean. The designs were built to theoretically have 95% confidence that the true mean was less than the calculated UCL, then the ideal result from the 2,000 iterations would be to find approximately 100 (5%) of the UCL values that were below the true mean. Figure 11 shows a histogram of 2,000 UCL values from this one simulation scenario where the y-axis represents the percentage of 2,000 in each bin (note that the y-axis is distorted to show the low bin counts). The red line identifies the location of the true mean. This UCL histogram shows that much less than 95% of the UCL values were above the true mean. Due to the coarsness of the histogram it is not clear what the exact coverage was, in fact the coverage was only 76%. Since this design was built to theoretically have 95% confidence then the simulation results from this exercise would show that the resulting coverage did not meet the design criteria and should not be used in practice.

Table 3. Example of UCL Simulations.

Mean	UCL
0.61	0.76
0.72	0.94
1.01	1.46

0.79	1.18
⋮	⋮
0.81	1.02
0.94	1.29

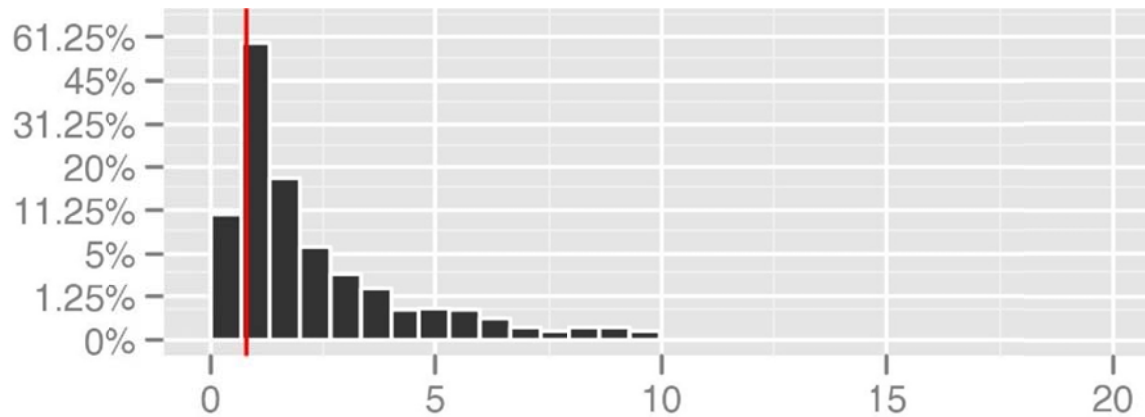

Figure 11. Histogram of the calculated UCL values using a simulated data set with 2,000 iterations. For display purposes the y-axis is in terms of percentage of 2,000 and is distorted (not evenly spaced between ticks) to highlight the low count bins. The red line identifies the true mean of 0.776.

Each of the different combinations of simulated sites and sampling patterns were evaluated in the manner described in this section. The following sub-sections will provide these results seperated by the five different simulated decision units – RDX, RDX 2, NG, 2-4 DNT, and NG. Within each simulated decision unit sub-section 40 sets of results will be shown using two different UCL calculation methods. We evaluated both systematic grid and random grid sampling routines for the grouped and standard IS patterns. The results for these methods were identical within the simulation error, thus only the random grid sampling routines are presented in each section for simplicity in displaying the results (see conclusions section for further detail). The simple random sampling results as compared to the random grid sampling routines are compared for the standard IS designs in the conclusions section.

The tables shown in each section are be seperated into the three general sampling patterns – standard IS, grouped IS, discrete sampling. Each table summarizes the results from 2,000 iterations. The first two columns are different for the IS and discrete summary tables. For the IS summary tables the first column identifies the number of IS sampled from within the DU and the second column shows the number of increments in each IS. For the discrete summary tables the first column identifies if random or systematic sampling was used and the second column lists the number of increments sampled from the DU that are used to calculate the mean and standard deviation. The third and fourth columns show the UCL coverage for the Chebyshev and t UCL calculations. The last four columns sumarize the relative percent difference (RPD) of the UCL values using the Chebyshev and t-distribution UCL multipliers. The “RPD Above ” columns for each UCL multiplier is the average relative difference of the UCL from the true mean for those UCL values that were above the true mean. The “RPD Below ” columns for each UCL multiplier show the average relative difference of the UCL from the true mean for those UCL values that were below the true mean.

Each sub-section contains plots depicting the pertinent information from the coverage tables for an easier visualization of the results from simulation studies. These plots show the designed UCL coverage level (dashed blue line) and the coverage performance of each sampling pattern as a function of the number of increments (in each IS for the IS designs and total for discrete designs). Each colored line

represents a different sampling pattern with the a separate plot for the discrete, grouped IS, and Standard IS. The dashed line identifies the t-UCL calculations and the solid line identifies the Chebyshev UCL values. Each plotted point represents the results from one line from the tables within the sub-section. Coverage results based on 2,000 iterations will provide accurate estimates to within approximately ±1.5% to ±2.5%.

One figure of 40 UCL histograms with consistent axes is shown in each sub-section. These figures are meant to show general distributional and coverage patterns of the calculated UCLs over all sampling patterns and may be difficult to use for evaluating any one histogram in the detail that can be done in Figure 11.

The displayed t-distribution UCL calculations are based on a 95% UCL using t-distribution with the degrees of freedom (*df*) equal to the number of measures used to calculate the standard deviation minus one for each scenario. For the IS sampling patterns *df* is the number of IS gathered from the site minus one. For the descrete sampling patterns df is the number of samples gathered minus one. We understand that the t-distribution is not appropriate for cases where the sample size is small and the measured values do not follow a normal distribution. This would generally be the case for the discrete sample designs with 9 and 16 samples as applied to the 5 simulated sites. In many instances a different UCL method would be needed for all discrete sample designs (16, 30, 49 and 100). Alternative UCL calculations that do not rely on normal theory should be used in those cases. Such UCL calculations can be found in software such as ProUCL (Schumacher, Singh, Maichle, Singh, & Lee) and Visual Sample Plan (VSP) (Matzke, Hassig, Wilson, Gilbert, & Pulsipher, 2007) for use in environmental studies. There are a variety of choices depending on site specific needs.

For the proposed IS sampling methods the t-distribution may not provide adequate coverage and with the limited number of available data values it is difficult to use many of the tools in ProUCL for alternative UCL calculations. Thus, a more conservative Chebyshev multiplier is used for attaining an improved coverage percentage. The UCL coverage plots and tables also show the Chebyshev 95% UCL calculations. The standard error is multiplied by a pre-specified value and added to the mean to identify the UCL. For the t-distribution this value is a function of the number of values used to estimate the mean and standard error. The Chebyshev multiplier is 1/sqrt(1-.95) for a 95% UCL regardless of the sample size used. This generally conservative multiplier of 4.472 will shift the coverage statistics up for all sampling patterns except for the 2 IS designs. As a t-distribution with one degree of freedom (df) results in a multiplier of 6.313. The most drastic effects of the Chebyshev multiplier are seen with the discrete designs as their coverage and bias increases the most.

NG 10 x 10 Decision Unit

For the NG 10 x 10 Decision unit Tables 4-6 show the the summaries from the evaluated simulations. The coverage, bias, number of increments, and number of IS were used to create the coverage plot shown in Figure 12. Figure 13 shows the panel of t-UCL histograms for all 40 sampling patterns evaluated on the NG 10 x 10 DU.

This site had some spatial patterns but was one of the least spatially heterogeneous sites of the 5 evaluated with a coefficient of variation (CV) of 1.2, where CV is the standard deviation (3.41) devided by the mean (2.83) of all increments from the DU. The coverage results for standard IS were at the designed criteria for designs with at least 49 increments included per IS and grouped IS was above the designed criteria of 95% for all designs. These results can be readily seen from the plots in Figure 12 where all grouped IS sampling patterns are above the designed 95% coverage line (dashed blue) and the standard IS designs with 49 and 100 increments were within the simulation error of the appropriate

coverage. As will be the case with the results for the following 4 scenarios, the grouped IS designs generally have greater coverage, but result in larger RPD values for those UCL values that are above the mean.

Table 4. Discrete summary: NG decision unit

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	98.30	91.05	171.90	78.82	15.77	17.70
Systematic	9	94.45	83.25	175.00	87.33	18.15	23.28
Random	16	99.60	94.35	130.33	54.52	10.03	11.16
Systematic	16	99.70	89.90	130.28	58.14	7.61	11.21
Random	30	99.95	94.55	93.98	37.50	13.69	7.69
Systematic	30	99.80	89.95	91.18	38.36	3.55	10.04
Random	100	100.00	97.35	53.79	20.67		3.63
Systematic	100	100.00	98.00	53.74	20.37		0.35

Table 5. Standard IS summary: NG decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	90.45	92.65	73.43	100.51	12.20	11.53
3	16	95.35	91.85	62.77	42.84	8.71	9.30
4	16	98.70	92.75	55.97	31.78	6.65	5.84
5	16	98.95	92.70	50.58	26.29	4.17	6.00
2	30	90.10	92.85	57.28	78.24	9.73	9.79
3	30	95.85	92.40	47.38	32.04	7.25	8.13
4	30	98.10	91.85	41.68	23.22	5.56	5.81
5	30	98.65	91.00	37.19	18.87	4.00	4.66
2	49	94.05	95.70	42.80	58.66	7.41	7.20
3	49	98.10	95.90	37.24	25.37	5.27	5.17
4	49	98.95	95.35	32.77	18.42	3.30	4.00
5	49	99.55	95.65	30.38	15.71	2.28	3.41
2	100	92.75	94.45	21.99	30.21	4.22	4.29
3	100	98.55	95.75	18.36	12.45	2.49	2.24
4	100	98.65	95.15	16.55	9.18	2.41	2.44
5	100	99.45	94.45	14.69	7.43	2.30	1.97

Table 6. Grouped IS summary: NG decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	93.90	95.75	101.81	141.11	9.78	9.15
3	16	98.75	96.65	84.22	56.05	7.16	7.28
4	16	100.00	100.00	176.57	93.79		
5	16	99.80	97.70	70.57	34.63	4.08	4.70
2	30	94.65	96.20	97.39	135.16	8.11	7.62
3	30	98.25	95.85	78.34	52.08	6.98	6.62
4	30	100.00	100.00	167.50	87.94		
5	30	99.80	97.90	64.27	31.22	2.59	4.50
2	49	99.10	99.40	89.18	124.97	2.68	2.40
3	49	99.80	98.90	71.24	46.46	2.70	2.47
4	49	100.00	100.00	172.14	91.05		
5	49	100.00	99.75	58.87	27.97		2.00
2	100	99.85	99.85	86.97	122.72	1.84	1.72
3	100	100.00	100.00	66.45	42.65		
4	100	100.00	100.00	168.39	88.60		
5	100	100.00	100.00	54.73	26.09		

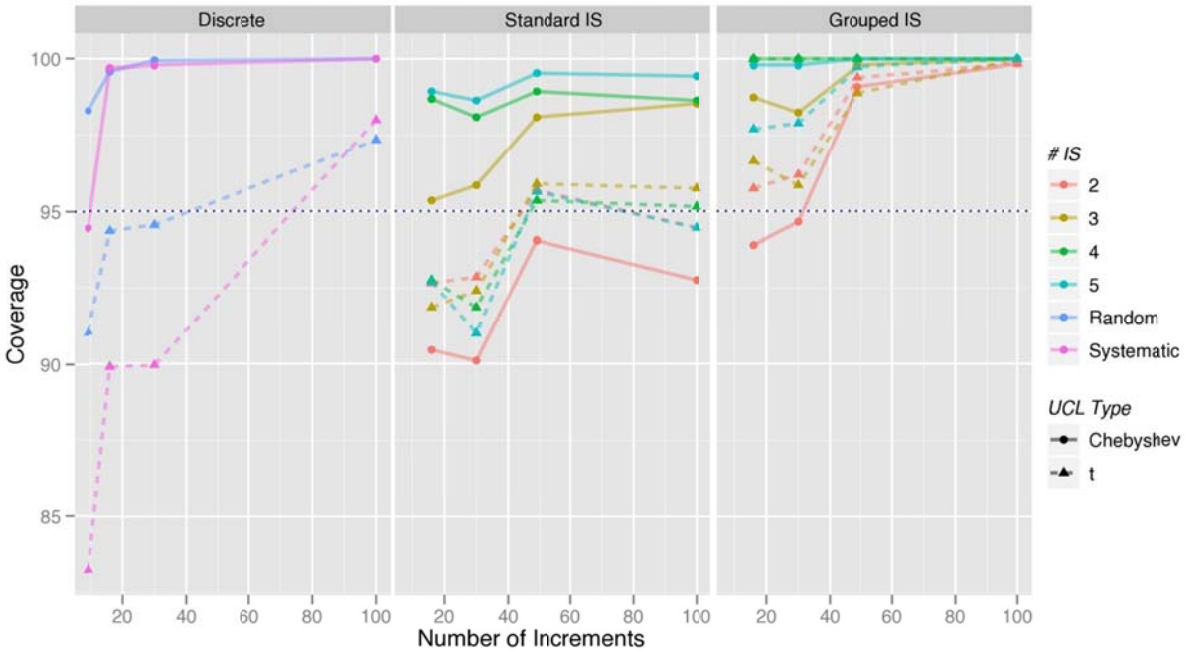

Figure 12. Plot of the coverage statistics for each of the simulated sampling patterns as applied to the NG decision unit. The different sampling patterns are displayed within the plot as well as the UCL type.

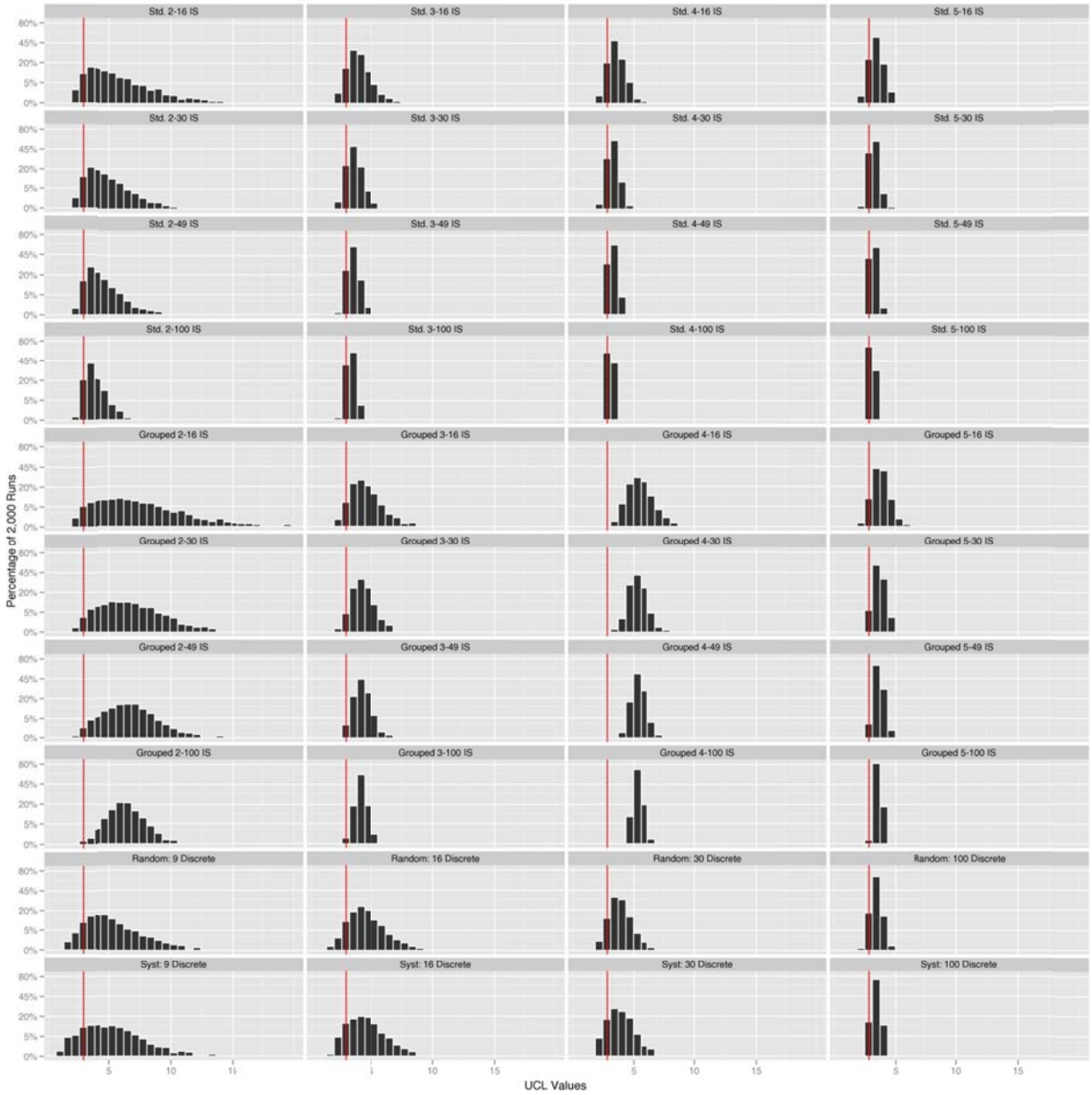

Figure 13. Panel of histograms of the distribution of t-UCL values for the 2,000 simulations. The red line identifies the true mean. The y-axis identifies the percent of 2,000 simulation in each bin and is distorted to show the percentage in the low count bins.

RDX 10 x 10 Decision Unit

For the RDX 10 x 10 Decision unit Tables 7-9 show the the summaries from the evaluated simulations. The coverage, bias, number of increments, and number of IS were used to create the coverage plot shown in Figure 14. Figure 15 shows the panel of t-UCL histograms for all 40 sampling patterns evaluated on the RDX 10 x 10 DU. This DU was used in the case study presented at the beginning of this report.

This site had the strongest small and large scale spatial heterogeneity of the 5 DUs evaluated with a CV of 4.47. The mean is 71.36 with a standard deviation of 319.1 . The coverage results for the standard IS perform reasonably well for the the IS designs of 100 increments per IS. The grouped IS patterns were above the designed criteria of 95% for all but the IS composed of 16 increments. For this DU the grouped IS are the only patterns that consistently met or exceded the designed 95% coverage but did have higher above mean RPDs than the standard IS or discrete methods.

Table 7. Discrete summary: RDX decision unit

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	67.20	55.80	596.67	334.23	57.02	61.88
Systematic	9	67.65	54.90	576.75	328.07	56.18	60.07
Random	16	79.25	64.50	431.13	229.60	45.61	49.98
Systematic	16	81.80	65.75	425.83	229.09	47.11	48.37
Random	30	84.60	67.75	292.69	145.30	34.17	40.99
Systematic	30	85.80	67.95	304.20	154.45	39.45	40.97
Random	100	97.50	84.50	182.32	81.15	13.70	20.02
Systematic	100	97.95	86.80	186.52	81.02	12.22	15.26

Table 8. Standard IS summary: RDX decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	82.95	86.35	279.99	373.67	37.86	36.14
3	16	88.15	81.95	219.34	157.50	27.98	30.40
4	16	92.35	82.25	199.60	122.60	24.52	26.07
5	16	94.00	82.45	177.73	99.96	20.89	22.80
2	30	82.35	86.70	192.10	257.12	31.80	31.52
3	30	90.50	83.90	150.90	105.86	23.31	24.57
4	30	93.65	83.95	135.61	78.51	20.59	21.45
5	30	95.85	82.95	119.96	64.14	16.60	17.27
2	49	87.85	90.55	147.00	200.34	25.16	23.89
3	49	93.20	88.30	128.19	89.26	16.46	17.75
4	49	96.45	88.40	111.83	64.84	15.40	15.31
5	49	96.85	88.90	101.49	53.30	14.40	15.13
2	100	88.10	91.10	100.46	136.07	16.05	16.26
3	100	94.80	90.80	85.38	59.17	9.62	11.27
4	100	97.60	92.70	76.04	43.07	7.87	10.39
5	100	98.30	91.70	67.41	35.27	8.17	7.79

Table 9. Grouped IS summary: RDX decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	90.55	93.00	408.76	560.08	41.12	42.40
3	16	94.90	90.75	380.21	261.11	31.43	31.17
4	16	95.75	88.45	277.75	159.09	21.51	25.65
5	16	97.95	92.50	297.41	152.63	17.43	23.42
2	30	96.05	97.85	372.63	516.93	29.51	34.85
3	30	98.90	96.15	334.96	223.28	21.55	19.55
4	30	99.35	95.65	239.38	128.93	13.45	18.42
5	30	99.80	96.20	267.77	131.54	13.41	14.40
2	49	99.75	99.95	375.05	528.31	8.90	3.84
3	49	100.00	100.00	342.29	222.02		
4	49	99.75	98.55	240.99	127.20	7.37	17.19
5	49	100.00	97.50	261.90	124.83		12.04
2	100	100.00	100.00	374.57	528.80		
3	100	100.00	100.00	336.40	217.67		
4	100	100.00	100.00	238.50	125.84		
5	100	100.00	100.00	266.15	126.93		

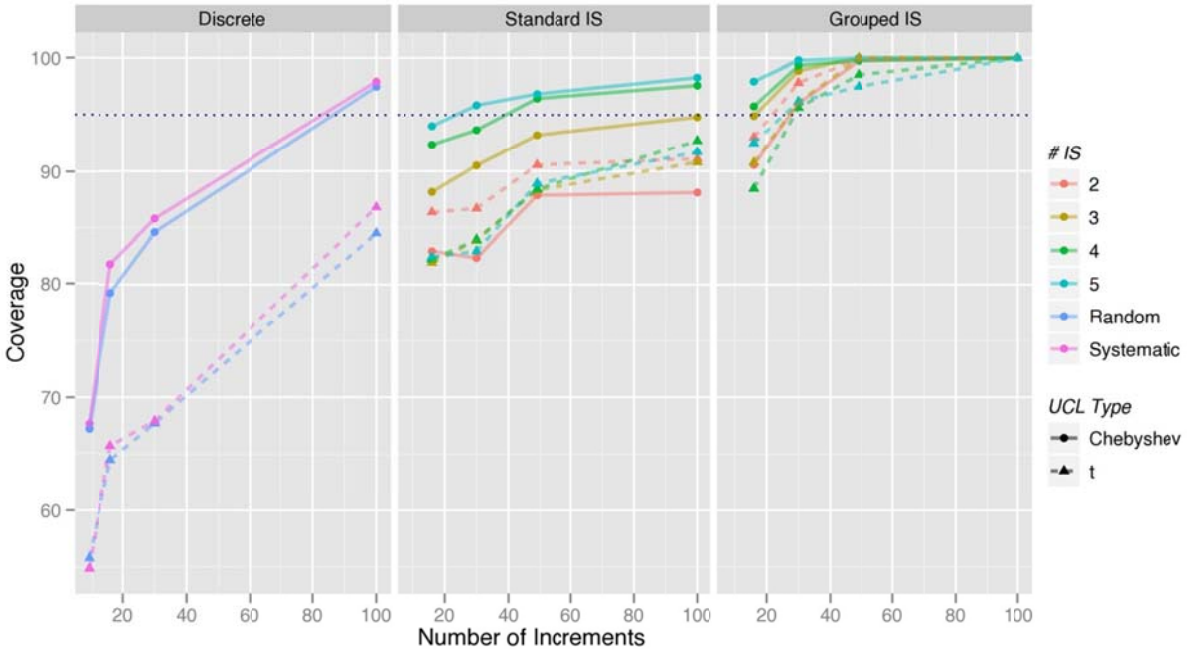

Figure 14. Plot of the coverage statistics for each of the simulated sampling patterns as applied to the RDX decision unit. The different sampling patterns are displayed within the plot as well as UCL type.

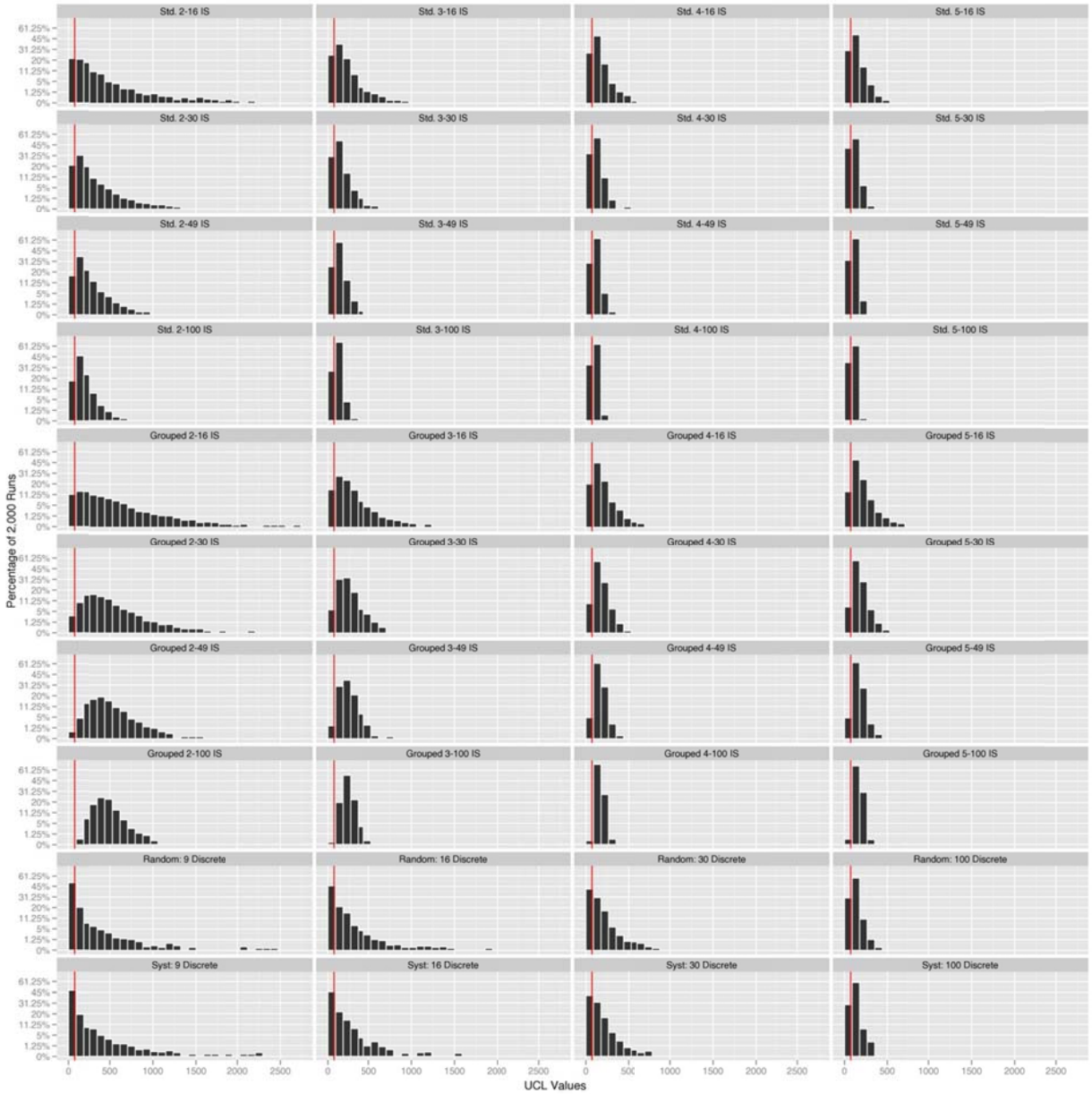

Figure 15. Panel of histograms of the distribution of t-UCL values for the 2,000 simulations. The red line identifies the true mean. The y-axis identifies the percent of 2,000 simulation in each bin and is distorted to show the percentage in the low count bins.

RDX 10 x 10 Decision Unit 2

For the RDX 10 x 10 Decision unit 2 Tables 10-12 show the the summaries from the evaluated simulations. The coverage, bias, number of increments, and number of IS were used to create the coverage plot shown in Figure 16. Figure 17 shows the panel of t-UCL histograms for all 40 sampling patterns evaluated on the RDX 10 x 10 DU 2.

This DU has some strong spatial heterogeneity, but the distribution of concentration values are not as skewed or heavily right tailed as the RDX DU with a CV of 2.27. The mean is 0.776 with a standard

deviation of 1.76. The coverage results for the standard IS identify reasonable coverage for only the IS designs which have 100 increments included. The grouped IS patterns were near or above the designed criteria of 95% for both the 49 and 100 increment IS with at least 3 replicates. For this DU the grouped IS consistently met or exceded the designed 95% coverage but did have more bias than the standard IS or discrete methods.

Table 10. Discrete summary: RDX decision unit 2

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	91.60	71.25	247.80	142.03	22.14	26.36
Systematic	9	91.85	72.90	255.48	144.00	22.42	25.82
Random	16	93.70	73.95	194.56	102.56	17.48	22.35
Systematic	16	95.40	75.50	196.30	104.19	15.67	20.51
Random	30	97.30	78.50	153.13	76.00	12.53	16.59
Systematic	30	97.10	78.15	155.50	77.90	8.37	16.88
Random	100	99.60	87.00	91.94	39.97	6.76	8.91
Systematic	100	98.10	83.60	95.15	43.11	3.83	8.65

Table 11. Standard IS summary: RDX decision unit 2

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	84.85	87.85	139.76	188.68	20.21	19.39
3	16	91.45	85.20	118.21	84.11	13.94	14.79
4	16	94.55	85.05	106.03	63.88	11.67	12.81
5	16	95.40	83.25	96.47	54.28	9.75	11.59
2	30	87.45	91.05	109.79	147.01	14.20	14.66
3	30	93.40	88.85	91.24	63.97	12.26	12.85
4	30	96.45	89.10	77.42	45.05	10.23	10.23
5	30	98.20	89.55	69.82	37.58	6.90	8.29
2	49	90.25	92.85	85.99	117.22	12.86	12.55
3	49	95.20	91.15	71.98	49.51	9.74	10.02
4	49	97.10	90.05	63.22	36.52	7.26	8.24
5	49	97.70	88.35	54.87	28.90	5.36	6.69
2	100	91.80	94.40	53.15	72.32	8.76	9.48
3	100	96.80	94.15	45.12	30.57	5.70	6.51
4	100	99.05	94.05	39.40	22.04	6.83	4.78
5	100	99.65	94.50	37.21	18.99	2.35	3.92

Table 12. Grouped IS summary: RDX decision unit 2

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	87.45	91.00	147.07	197.48	17.59	17.73

3	16	93.40	87.30	129.90	92.21	12.64	13.24
4	16	99.05	94.95	144.83	79.57	10.21	10.07
5	16	97.65	86.75	115.06	63.55	8.79	10.70
2	30	87.45	91.05	116.68	156.87	15.07	15.14
3	30	95.15	90.70	109.01	75.78	12.34	12.13
4	30	99.75	97.85	137.57	74.23	4.90	6.31
5	30	99.30	93.10	100.66	51.65	5.00	7.06
2	49	90.00	93.20	85.72	116.78	11.35	11.70
3	49	96.85	93.80	101.35	69.53	9.38	9.32
4	49	99.75	98.80	128.29	67.80	3.21	9.17
5	49	99.45	95.65	100.68	49.58	4.49	7.33
2	100	88.25	91.80	68.54	92.33	6.93	6.96
3	100	100.00	100.00	87.79	57.74		
4	100	100.00	100.00	123.64	65.07		
5	100	100.00	99.25	87.41	42.00		2.15

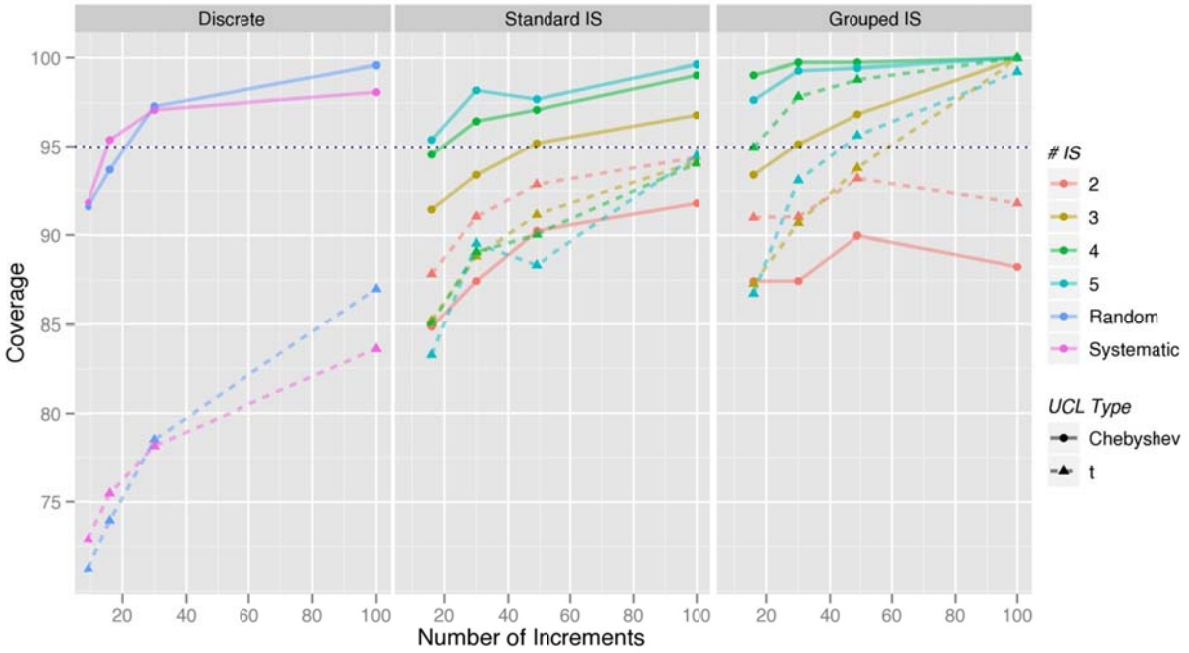

Figure 16. Plot of the coverage statistics for each of the simulated sampling patterns as applied to the RDX decision unit 2. The different sampling patterns are displayed within the plot as well as UCL type.

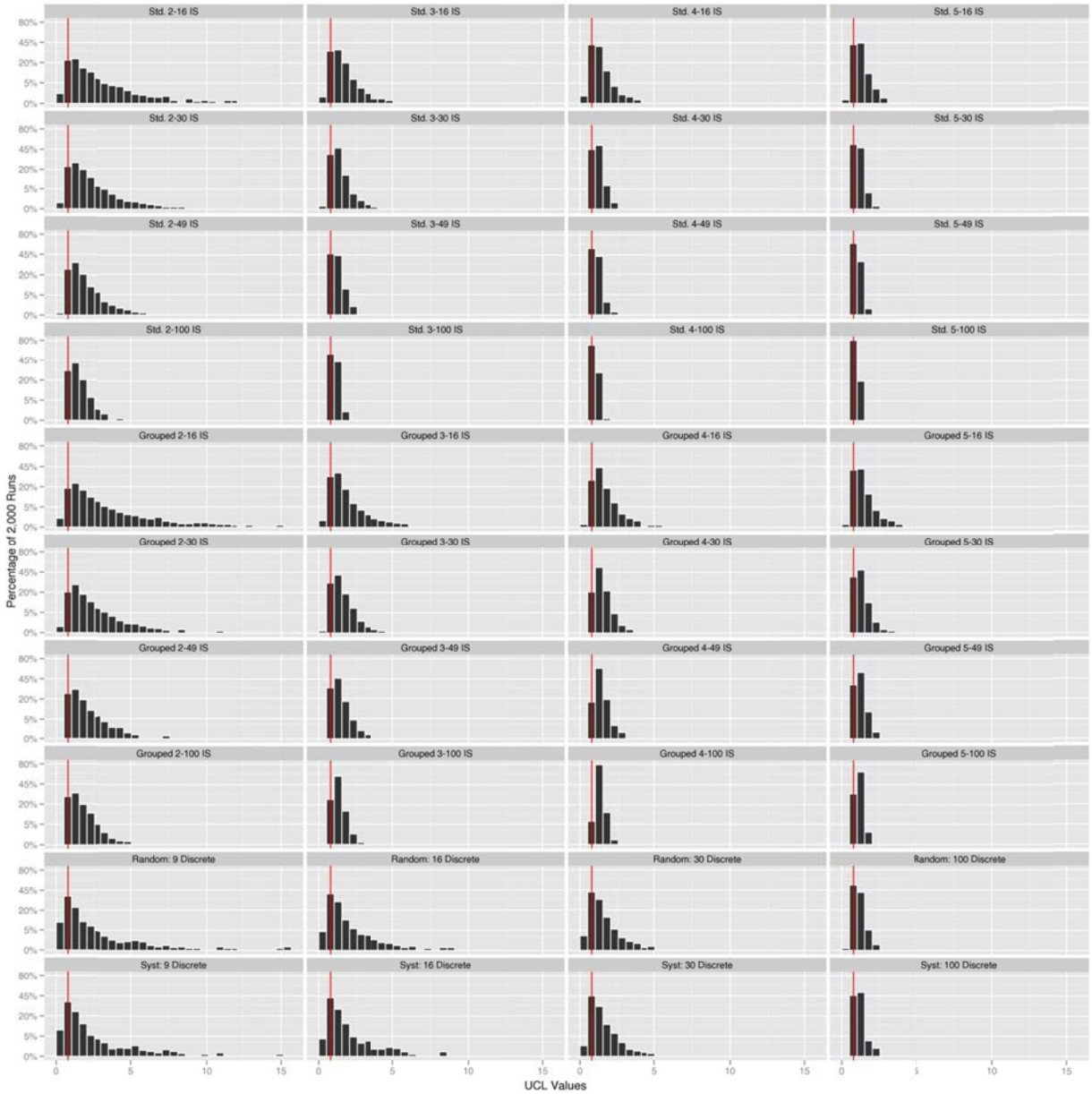

Figure 17. Panel of histograms of the distribution of t-UCL values for the 2,000 simulations. The red line identifies the true mean. The y-axis identifies the percent of 2,000 simulation in each bin and is distorted to show the percentage in the low count bins.

2-4 DNT 10 x 10 Decision Unit

For the 2-4 DNT 10 x 10 Decision unit , Tables 13-15 show the the summaries from the evaluated simulations. The coverage, bias, number of increments, and number of IS are used to create the coverage plot shown in Figure 18. Figure 19 shows the panel of t-UCL histograms for all 40 sampling patterns evaluated on the 2-4 DNT 10 x 10 DU.

This DU has some strong small scale spatial heterogenaity, but the distribution of concentration values are not as skewed or heavily right tailed with a CV of 1.69. The mean is 0.694 with a standard deviation

of 1.17. The coverage results for the standard IS come close to the desired coverage with the 100 increment IS designs. The grouped IS patterns were near or above the designed criteria of 95% for all designs with 49 or more Increments included.

Table 13. Discrete summary: 2-4 DNT decision unit

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	93.75	78.35	210.13	110.13	21.59	24.84
Systematic	9	93.55	79.20	211.76	109.23	20.81	24.41
Random	16	97.35	81.10	168.11	83.40	16.70	18.66
Systematic	16	96.55	81.45	170.28	84.75	16.04	21.70
Random	30	98.90	84.40	124.05	56.86	11.45	12.50
Systematic	30	98.65	82.10	127.02	60.80	13.43	12.49
Random	100	99.75	91.15	73.19	30.71	4.83	7.67
Systematic	100	100.00	93.55	73.01	29.01		2.78

Table 14. Standard IS summary: 2-4 DNT decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	88.60	91.45	108.65	147.53	14.80	14.18
3	16	94.60	89.05	92.47	64.95	9.36	11.07
4	16	96.50	88.25	83.01	48.48	9.23	10.18
5	16	97.85	89.20	75.74	40.35	6.59	8.54
2	30	88.85	91.40	89.69	122.33	13.03	12.16
3	30	94.35	88.70	74.03	51.81	8.72	9.38
4	30	96.30	88.55	64.20	37.26	5.55	7.93
5	30	97.35	87.60	56.95	30.38	5.57	7.55
2	49	89.80	92.50	62.61	85.34	9.25	8.79
3	49	96.55	91.85	51.79	35.85	6.24	6.09
4	49	98.20	91.70	46.48	26.52	4.32	5.30
5	49	98.80	91.85	41.69	21.47	2.73	4.69
2	100	89.90	92.35	42.68	57.92	7.01	6.81
3	100	94.90	91.25	34.90	23.99	4.96	5.37
4	100	97.50	92.05	31.01	17.72	4.13	4.20
5	100	99.00	92.95	27.66	14.35	2.94	3.13

Table 15. Grouped IS summary: 2-4 DNT decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	89.90	93.10	127.46	172.52	14.07	13.87
3	16	95.90	92.80	108.43	74.11	13.74	14.23
4	16	99.70	98.20	135.47	72.72	10.87	9.57

5	16	98.70	89.90	83.85	44.89	8.88	8.65
2	30	87.95	91.45	111.64	150.58	13.10	12.79
3	30	95.85	91.75	89.18	61.41	10.02	10.15
4	30	99.90	98.80	122.58	65.01	5.75	5.50
5	30	98.45	92.60	72.07	37.40	5.23	7.16
2	49	95.00	97.05	88.77	122.10	7.88	7.83
3	49	98.65	96.85	77.21	51.82	6.11	6.25
4	49	100.00	100.00	120.72	63.68		
5	49	99.60	96.30	63.89	32.13	4.40	5.03
2	100	97.35	97.85	77.22	108.42	6.53	5.86
3	100	100.00	100.00	64.50	41.96		
4	100	100.00	100.00	115.06	60.52		
5	100	100.00	99.75	48.41	23.14		1.45

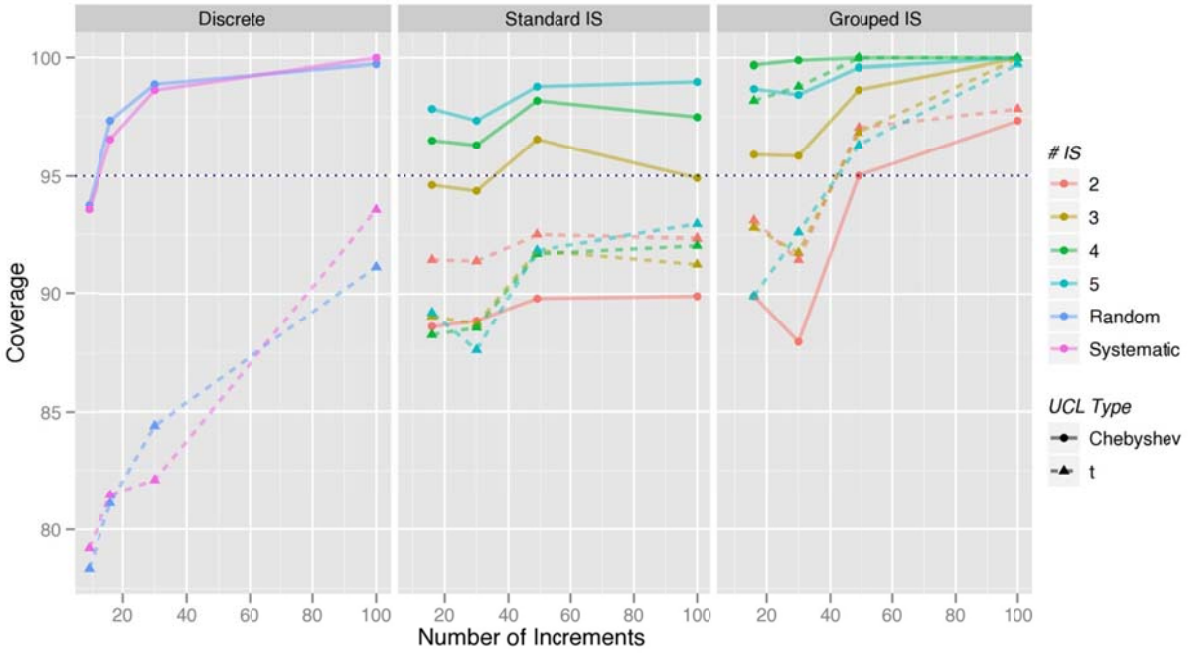

Figure 18. Plot of the coverage statistics for each of the simulated sampling patterns as applied to the 2-4 DNT decision unit. The different sampling patterns are displayed within the plot as well as UCL type.

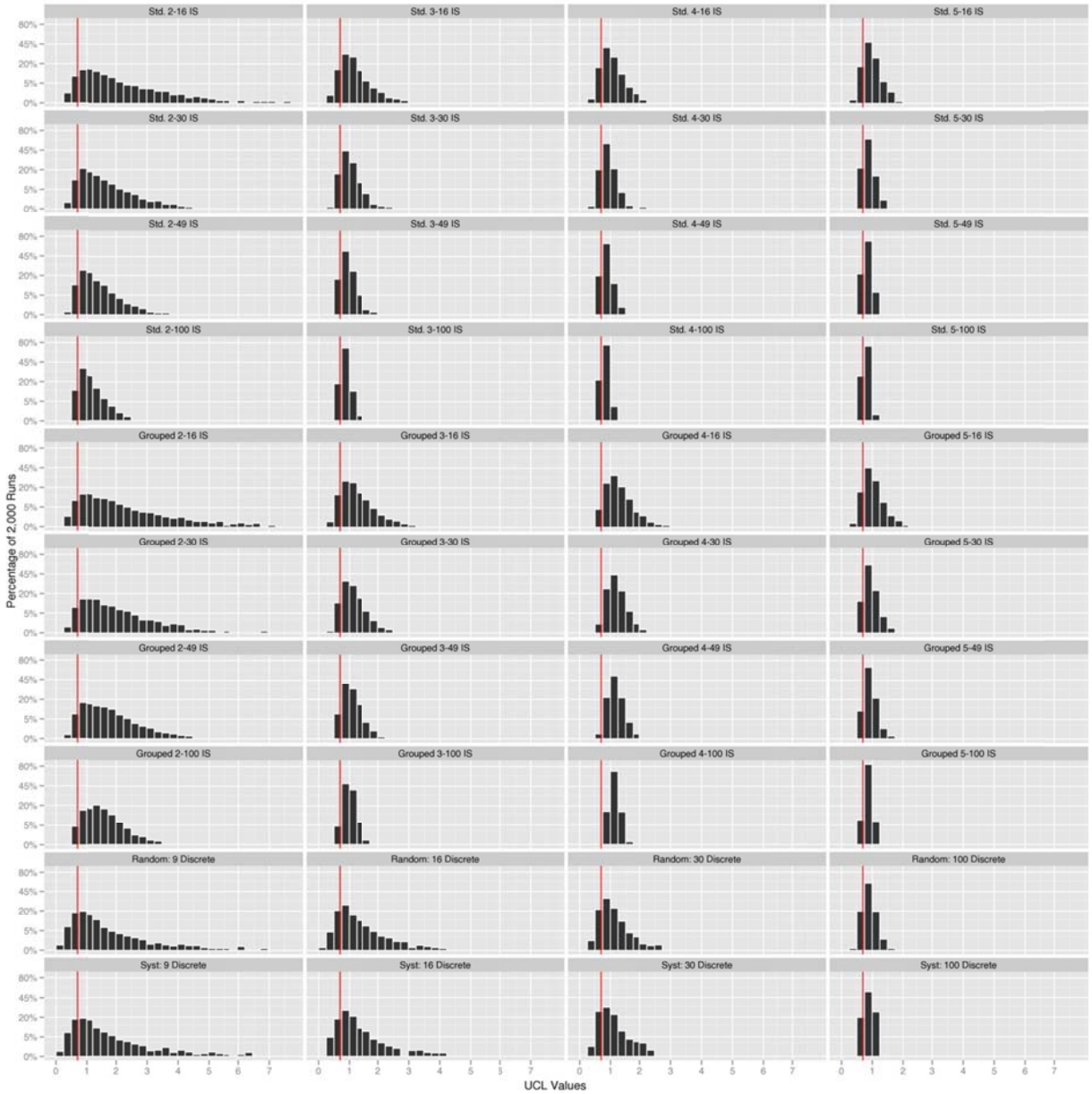

Figure 19. Panel of histograms of the distribution of t-UCL values for the 2,000 simulations. The red line identifies the true mean. The y-axis identifies the percent of 2,000 simulation in each bin and is distorted to show the percentage in the low count bins.

HMX 10 x 10 Decision Unit

For the HMX 10 x 10 Decision unit, Tables 16-18 show the the summaries from the evaluated simulations. The coverage, bias, number of increments, and number of IS are used to create the coverage plot shown in Figure 20. Figure 21 shows the panel of t-UCL histograms for all 40 sampling patterns evaluated on the HMX 10 x 10 DU.

This DU has some strong spatial heterogeneity, but the distribution of concentration values are not as skewed or heavily right tailed with a CV of 1.1. The mean is 0.132 with a standard deviation of 0.146.

When 3 or more replicates are used, the coverage results for the grouped IS patterns were near or above the designed criteria of 95% for all but the IS composed of 16 increments. The standard IS performed reasonably well for the 100 increment standard IS design.

Table 16. Discrete summary: HMX decision unit

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	97.65	85.55	140.04	69.15	13.56	15.08
Systematic	9	97.40	83.70	138.07	69.42	11.88	15.48
Random	16	99.05	87.00	110.39	51.50	6.36	11.33
Systematic	16	98.75	86.40	108.63	50.69	5.54	13.01
Random	30	99.55	87.45	83.39	37.49	4.61	8.02
Systematic	30	100.00	90.30	82.82	35.91		6.67
Random	100	100.00	92.20	48.01	19.73		4.15
Systematic	100	100.00	92.80	47.63	19.61		4.64

Table 17. Standard IS summary: HMX decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	90.70	93.60	69.45	94.49	9.84	10.13
3	16	96.50	92.20	59.36	41.10	8.33	7.33
4	16	97.55	90.10	52.77	30.81	5.48	6.15
5	16	98.85	91.75	47.53	24.89	2.97	4.66
2	30	90.20	92.75	50.93	68.96	6.64	6.10
3	30	96.15	92.85	40.70	27.96	5.22	5.69
4	30	98.20	94.00	36.95	20.86	3.59	4.59
5	30	98.75	92.35	33.07	17.44	3.89	3.90
2	49	90.30	92.85	39.87	54.62	6.10	6.01
3	49	96.40	92.35	34.71	23.86	4.60	4.47
4	49	97.65	91.90	29.76	16.71	2.97	3.68
5	49	98.95	92.50	26.96	13.85	2.55	3.56
2	100	91.40	93.30	28.15	38.71	4.67	4.69
3	100	96.95	93.60	22.86	15.56	3.77	3.41
4	100	98.65	94.15	20.29	11.39	1.55	2.27
5	100	99.10	94.05	18.50	9.47	2.10	2.24

Table 18. Grouped IS summary: HMX decision unit

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	90.85	92.85	70.55	96.55	9.55	8.54
3	16	96.95	93.05	61.46	42.15	6.42	6.05
4	16	99.80	98.95	89.73	47.36	2.40	5.87

5	16	99.15	93.65	52.03	26.73	5.65	5.19
2	30	91.20	94.00	50.99	69.54	6.84	7.05
3	30	98.55	95.95	47.58	32.33	2.99	3.93
4	30	100.00	99.75	87.32	46.18		3.89
5	30	99.45	95.25	38.94	19.40	4.19	3.44
2	49	92.05	95.35	38.57	52.55	5.02	5.78
3	49	98.70	96.75	38.65	26.05	3.67	3.43
4	49	100.00	100.00	83.60	43.78		
5	49	99.90	97.90	33.76	16.12	2.76	2.63
2	100	93.50	95.00	29.49	40.74	5.66	5.22
3	100	99.20	97.15	27.85	19.02	2.88	2.07
4	100	100.00	100.00	81.47	42.85		
5	100	99.75	98.95	26.67	12.83	1.97	2.38

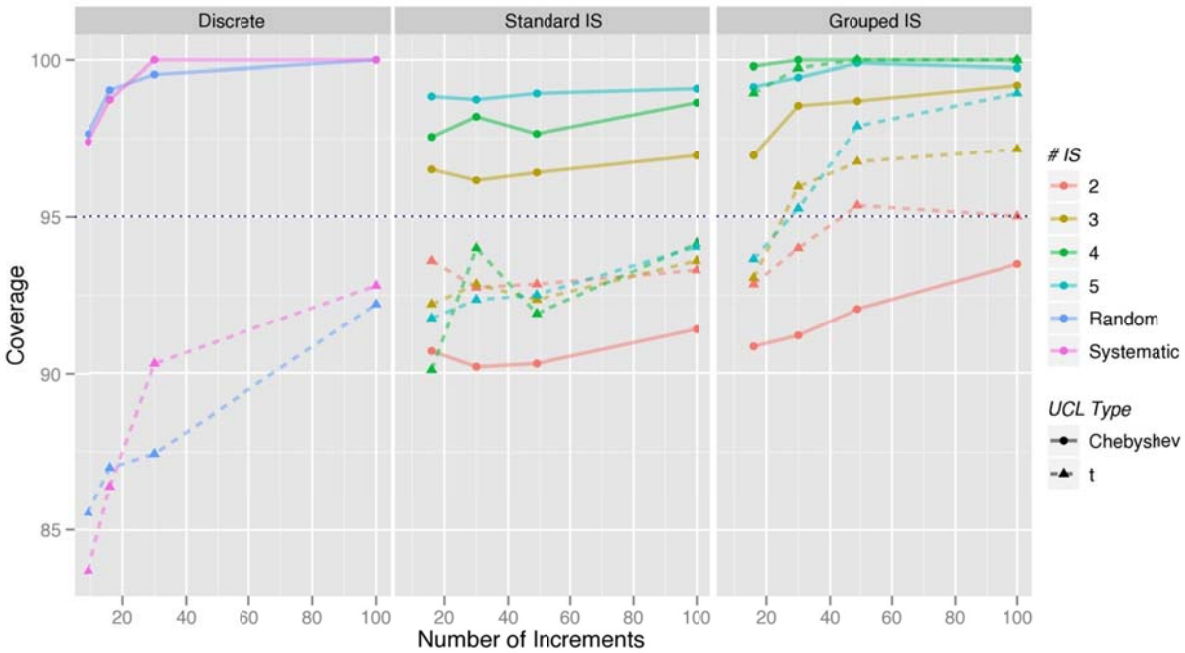

Figure 20. Plot of the coverage statistics for each of the simulated sampling patterns as applied to the HMX decision unit. The different sampling patterns are displayed within the plot as well as UCL type.

Figure 21. Panel of histograms of the distribution of t-UCL values for the 2,000 simulations. The red line identifies the true mean. The y-axis identifies the percent of 2,000 simulation in each bin and is distorted to show the percentage in the low count bins.

Conclusions

The five simulated sites were selected to represent a range of the more extreme spatial and concentration values (see Figures 2, 4 & 5). All 5 examples had concentration values that were heavily right tailed. The severity of the spatial patterns (elevated regions) varied for both the large and small scale heterogeneity. The RDX decision unit used in the case study had the most severe heterogeneity and the NG or HMX decision units were the least severe.

The reasonable performance under these extreme cases suggests that a t-UCL will perform quite well for decision units where the constituent of interest is expected to be less heterogeneous than those sites used in this simulation study. The Chebyshev UCL does improve coverage and should be used when the constituents of interest are believed to be severely heterogeneous. However, using the t-UCL appears to be acceptable when each IS is composed of 100 increments, based on the acceptable coverage statistics and smaller RPD achieved for all cases.

Some general conclusions can be made from the results shown in Figures 12,14,16,18, and 20. For sites with heavy right tailed distributions and spatial heterogeneity, discrete sampling methods with up to 100 samples taken are not sufficient to use a t-statistic to calculate a reliable UCL. However, the Chebyshev UCL does provide adequate coverage for many of the decision units at multiple sample sizes. Other methods for UCL calculations can be used to attain the accurate coverage (see documentation for ProUCL). When comparing the two IS methods (grouped and standard) the following results are pertinent.

- The mean concentration estimates for grouped IS and standard IS sampling have the same expectation and distribution (see Figure 22).
- The grouped IS methods have equivalent or greater coverage than standard IS when the same number of IS and increments are used.
- The UCL values for grouped IS are generally higher than UCL values using standard IS.
- Grouped IS by its definition will provide an improved spatial picture of the concentrations within the site.

Upon evaluating the different comparisons among the standard IS sampling patterns. Using these 5 simulated sites, the standard IS sampling approach with a t-distribution based UCL will meet the designed criteria when 100 increments are included in as few as two IS. As few as 30 increments can be included in each IS for DUs with less severe heterogeneity and will still attain reliable coverage (see NG DU). A list of other pertinent results follows.

- Systematic grid, Random grid, or SRS all generally give the same results in terms of coverage and the use of one or the other can be selected for ease of application (see Figure 23)
- For IS designs with more than 2 IS, the Chebyshev method (or some other non-parametric approach) may be necessary to attain adequate coverage depending on the severity of the heterogeneity.
- When 100 increments are included in an IS a t-distribution UCL will generally provide adequate coverage.
- The improvements in coverage are the most dramatic when the number of increments is increased as compared to increasing the number of IS.

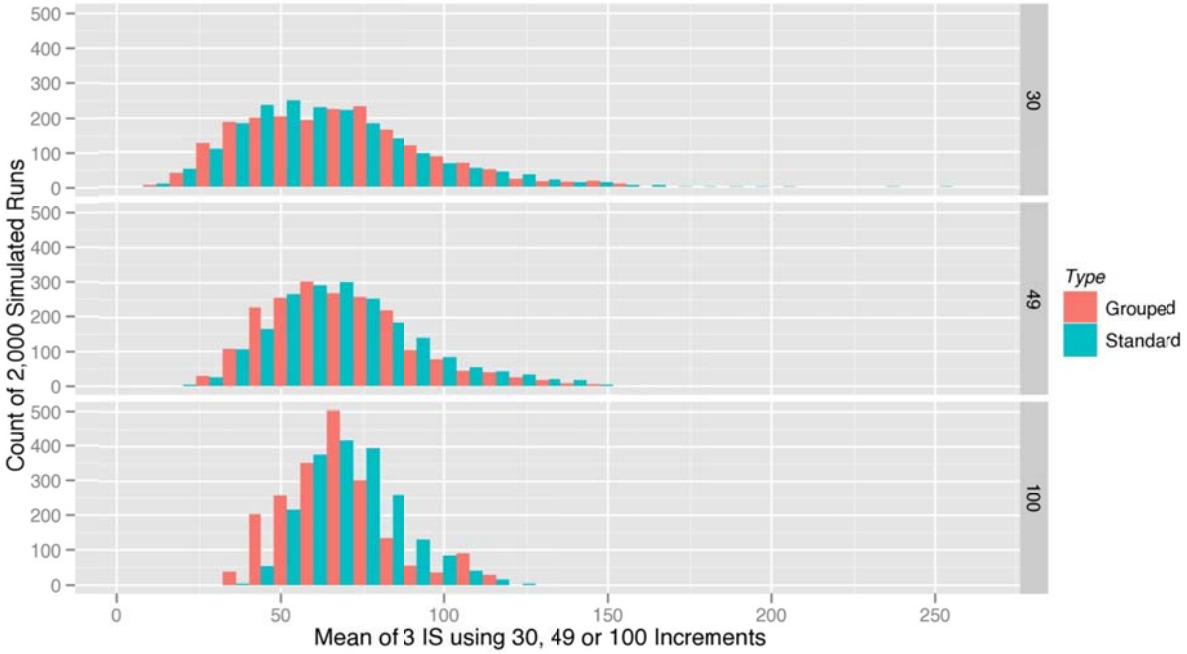

Figure 22. A comparison of the distribution of means for grouped and standard IS designs using the RDX DU. Results are similar for all other DUs.

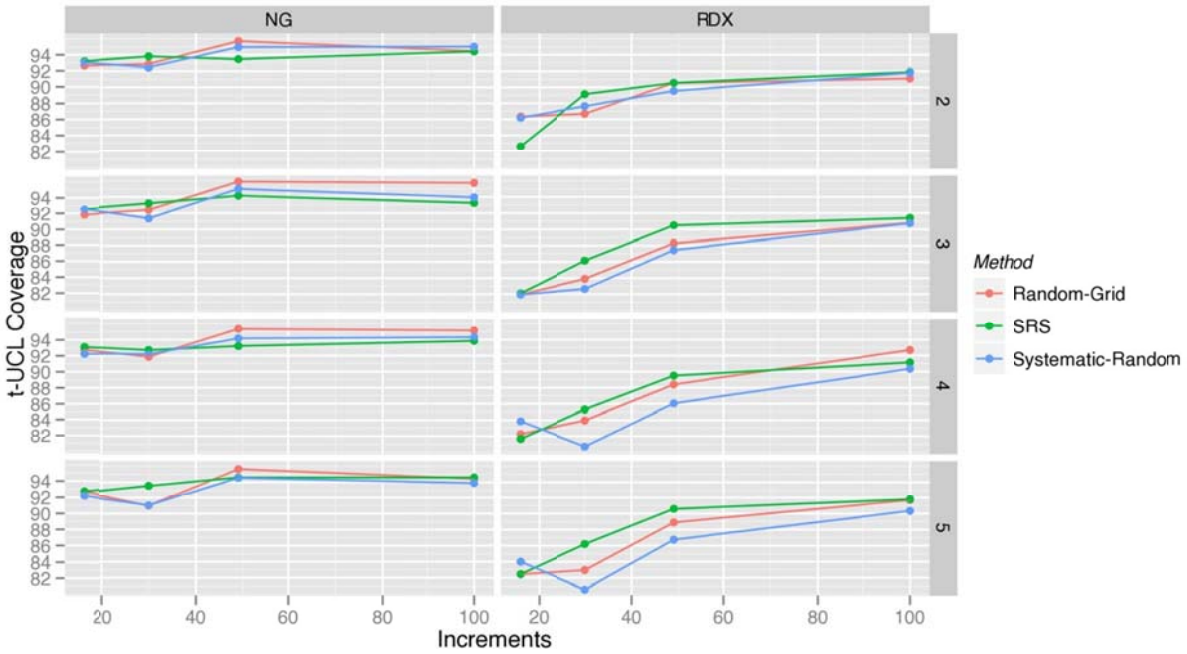

Figure 23. A coverage plot comparing systematic grid, random grid, and SRS for the RDX DU and NG DU when 2, 3, 4 or 5 IS are collected from the DU. The simulation uncertainty at each point estimate is between $\pm$1.5% to $\pm$2.5%.

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Appendix A – Simulated Site Development Documentation

Each of the five sites were developed from discrete data gathered during an evaluation of Incremental sampling under a demonstration project funded by SERDP/ESTCP (Hewitt, Jenkins, Ramsey, & Bjella, 2005; Jenkins, Hewitt, Ranney, & Ramsey, 2004; Jenkins, Thiboutot, Ampleman, & Hewitt, 2005). . All but the RDX DU are based on data obtained from CFB Petawawa, Canada. The RDX DU is located in Fort Polk, Al.

CFB Petawawa is located in the Ottawa Valley in Renfrew County, 170 kilometers (106 mi) northwest of Ottawa along the western bank of the Ottawa River. Its main gate is adjacent to the town of Petawawa. The majority of the base territory is in the municipality of Laurentian Hills, with portions also in Petawawa and Deep River. The base has an extensive infrastructure with 465 buildings and over 300 km^2 of property comprising the Petawawa Training Area.

Fort Polk is a United States Army base located near Leesville, Louisiana. It serves primarily as a training post for units preparing to deploy to fight the nation's wars. The main post consists of about 100,000 acres, mostly in the Kisatchie National Forest. It is the only Combat Training Center that also trains and deploys combat units and began as a base for the Louisiana Maneuvers in the 1940s.

Each of the three locations was evenly divided into a square grid with 100 cells. Allan and Tom gathered both incremental samples and discrete samples for each grid. The figures below show the measured values for each discrete sample taken over the 10x10 cell grid. Four different constituents are represented in the following figures (2,4-DNT, RDX, NG, HMX) and are labeled appropriately. The following figures provide examples of the basis for the simulated sites used for the simulation studies performed as a part of this report.

We have assumed that this discrete data provides a reasonable spatial understanding of the types of patterns that could be identified for munitions related constituents within DUs related to munitions constituents. We have also assumed that the small scale spatial variability can be estimated using the variance between increments that are spatially next to each other.

Figure A-1 shows a spatial map of the 100 measured RDX values from the samples within the 20 x20 m decision unit. These values were used to estimate a mean and standard deviation at each node. This process was done by systematically moving between nodes. At each node, the sample and all other samples within one node were used to estimate the mean and standard deviation. These means and standard deviations are shown in Figure A-2. This process was used to maintain both spatial and distributional properties for the simulated DUs that were similar to the real data gathered from within the DU. Figure A-3 shows the actual Gamma distributions used for nodes 10-5 and 1-8 (row, col). Node 1-8 had a mean and standard deviation of 0.89 and 0.67 which resulted in shape and scale parameters of 1.74 and 1.96 to define the Gamma distribution in this node. Node 10-5 had a mean and standard deviation of 864.9 and 988.8 that resulted in a shape and scale parameters of 0.77 and 0.0009. This similar process was used for the other 4 DUs used in this report. Figures A-4 through A-11 show the spatial maps and node parameter maps for the other 4 DUs.

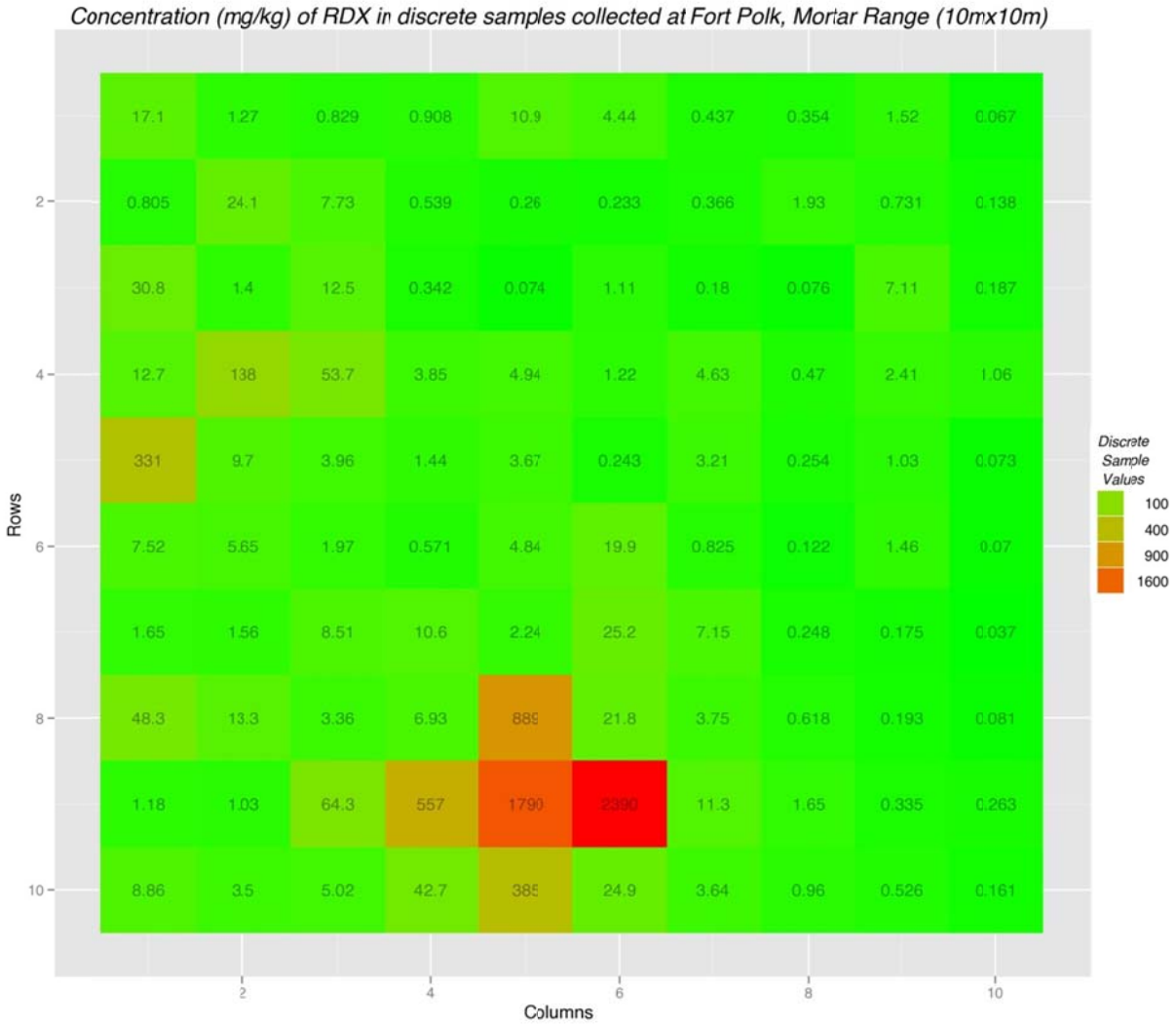
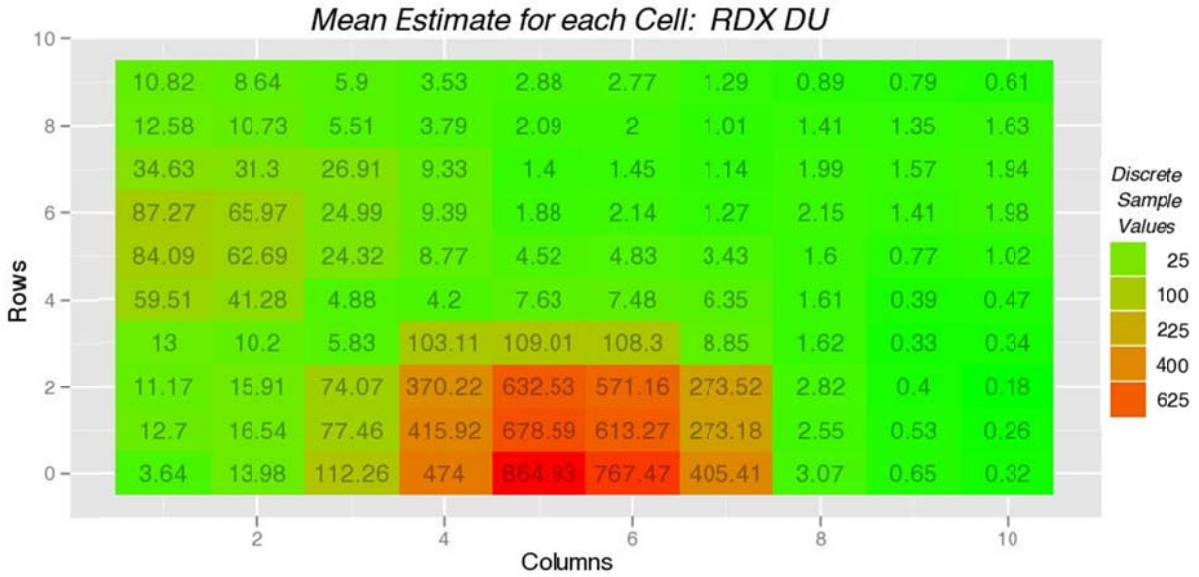


Figure A-1. Discrete data gathered from Fort Polk, AL within a 20 x 20 m decision unit were a discrete sample was collected from each node of a 10 x 10 grid. All RDX values are in mg/kg.



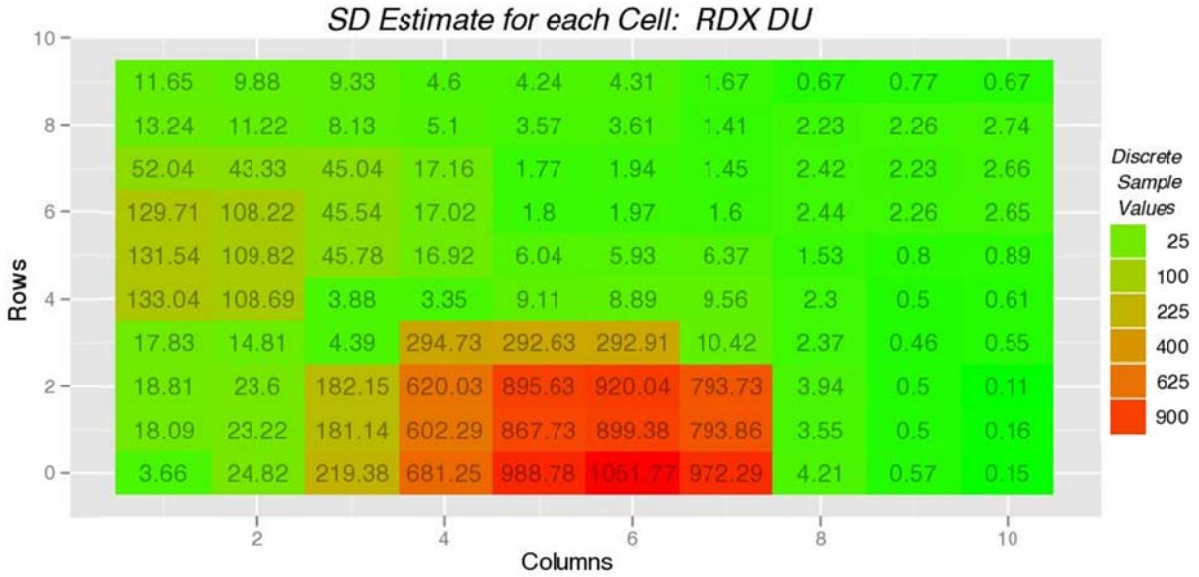


Figure A-2. Spatial mean and standard deviation estimates based on the discrete samples obtained during a study at Fort Polk (see Figure A-1). These values are used to identify the parameters for a unique Gamma distribution for each 1x1 m cell.

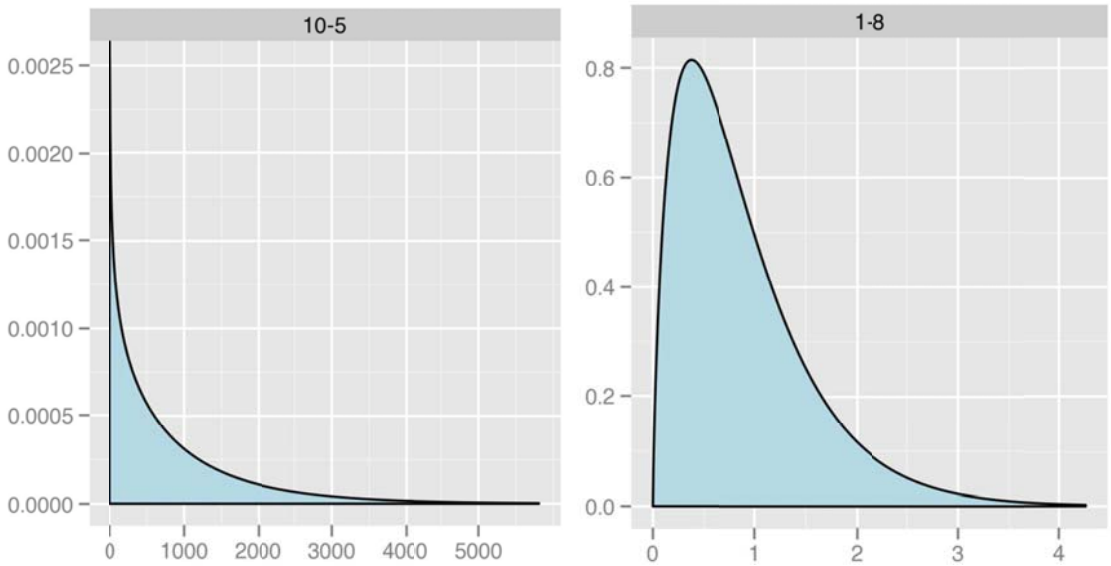
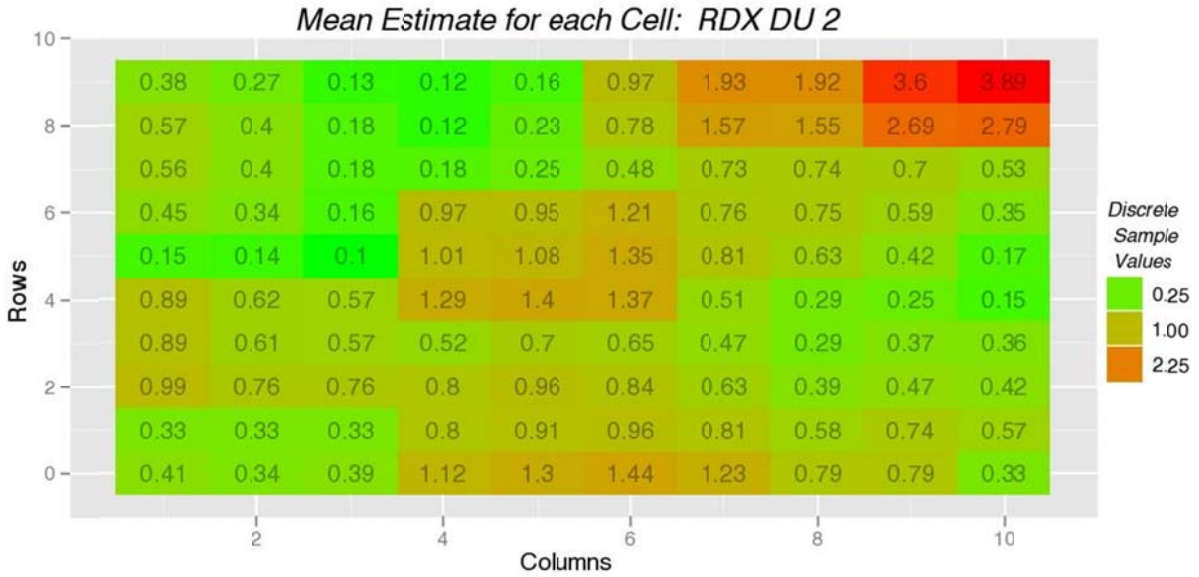


Figure A-3. Gamma distributions assumed for node 10,5 and 1,8 (row,col) of the RDX DU.

Figure A-4. Discrete data gathered from a Hand Grenade Range at CFB Petawawa, Canada within a 10 x 10 m decision unit were a discrete sample was collected from each node of a 10 x 10 grid. All RDX values are in mg/kg.



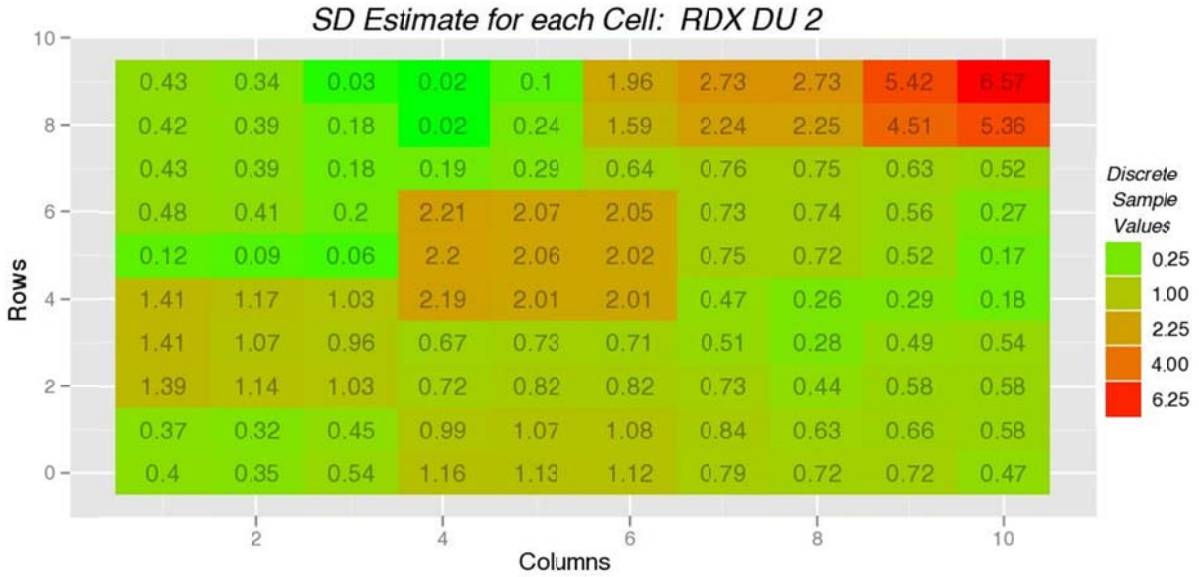


Figure A-5. Spatial mean and standard deviation estimates based on the discrete samples obtained during a study at CFB Petawawa, Canada (see Figure A-4). These values are used to identify the parameters for a unique Gamma distribution for each 1x1 m cell.

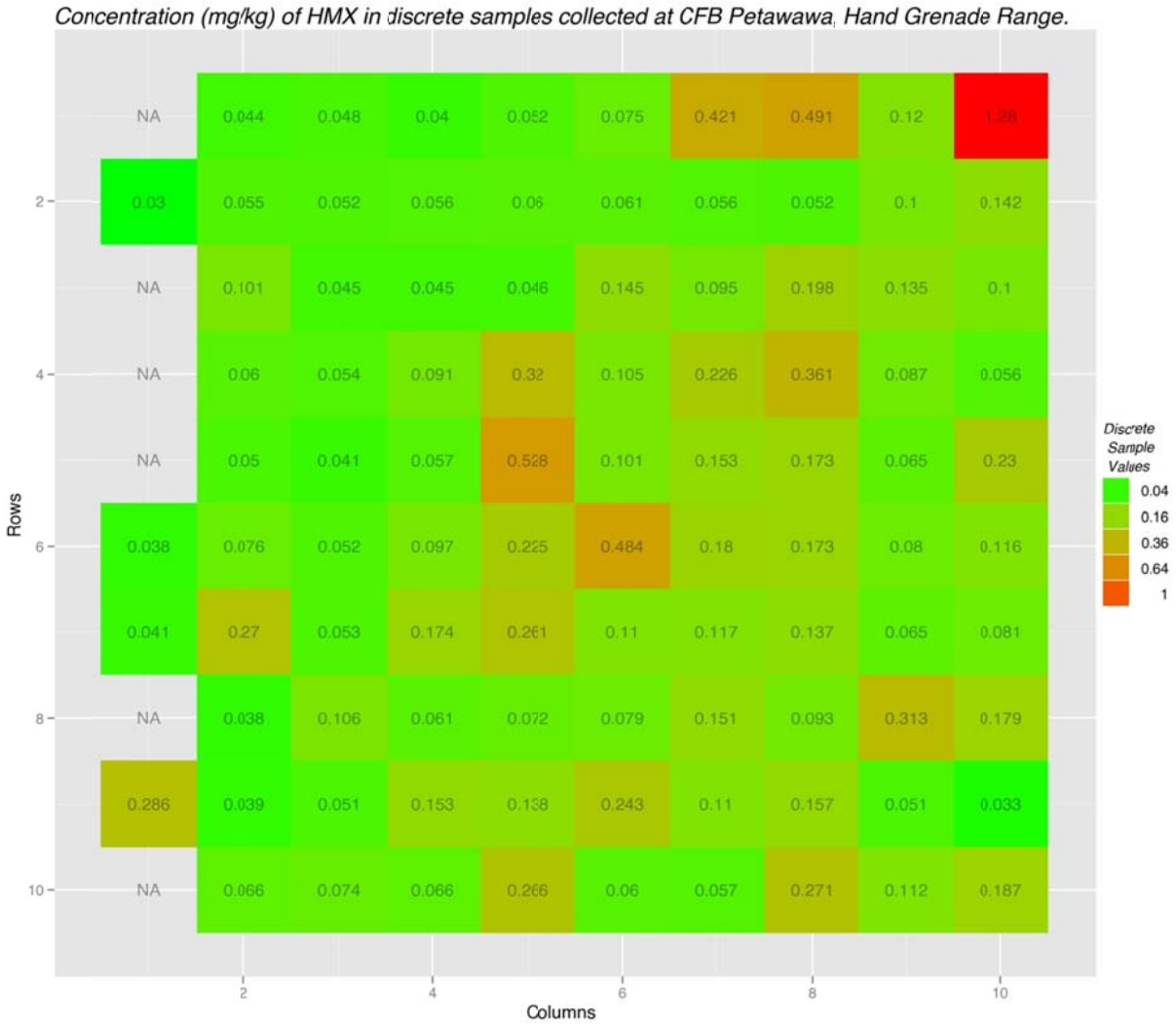


Figure A-6. Discrete data gathered from a Hand Grenade Range at CFB Petawawa, Canada within a 10 x 10 m decision unit were a discrete sample was collected from each node of a 10 x 10 grid. All HMX values are in mg/kg.

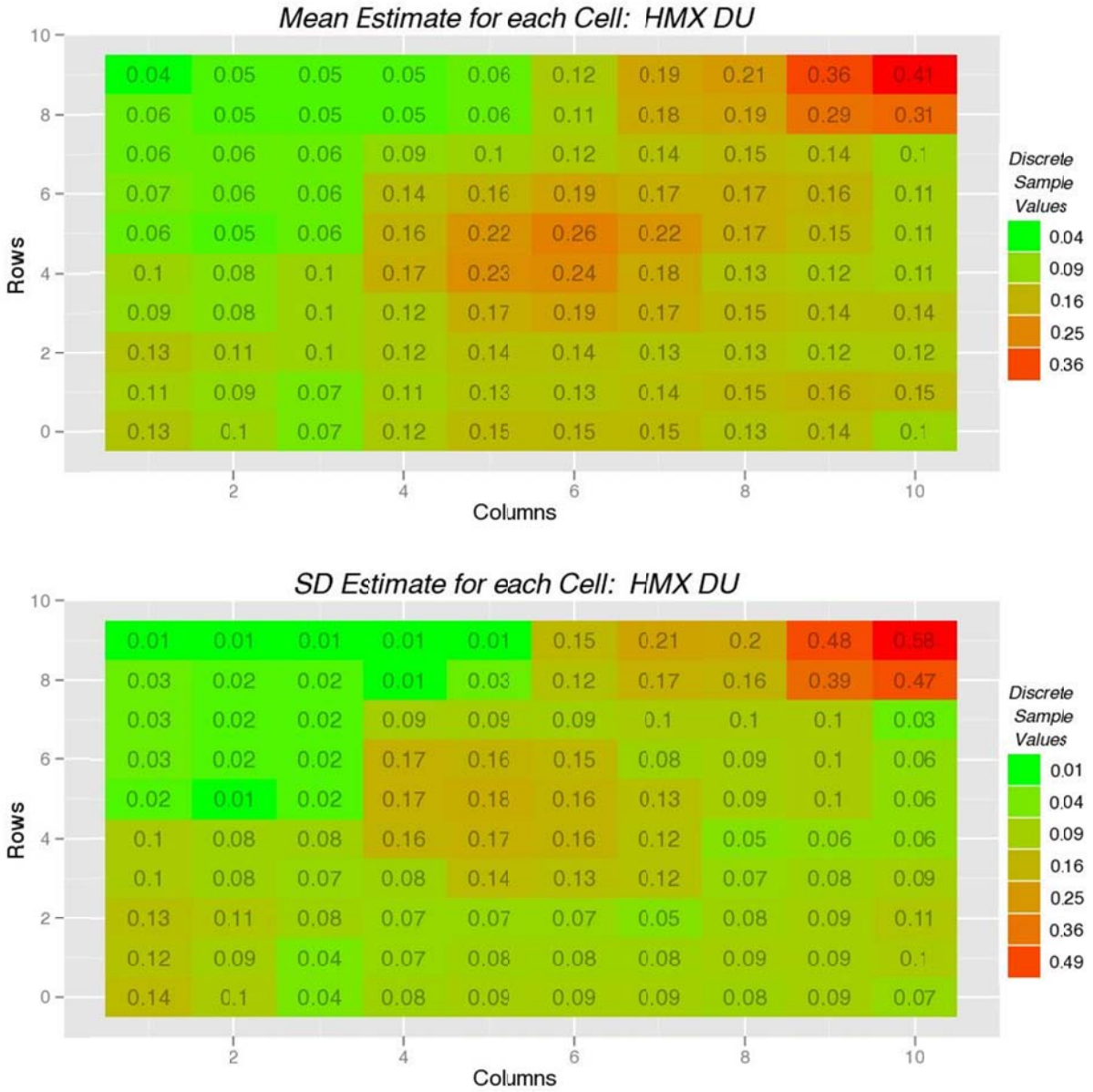


Figure A-7. Spatial mean and standard deviation estimates based on the discrete samples obtained during a study at CFB Petawawa, Canada (see Figure A-6). These values are used to identify the parameters for a unique Gamma distribution for each 1x1 m cell.

Figure A-8. Discrete data gathered from a Hand Grenade Range at CFB Petawawa, Canada within a 10 x 10 m decision unit were a discrete sample was collected from each node of a 10 x 10 grid. All NG values are in mg/kg.

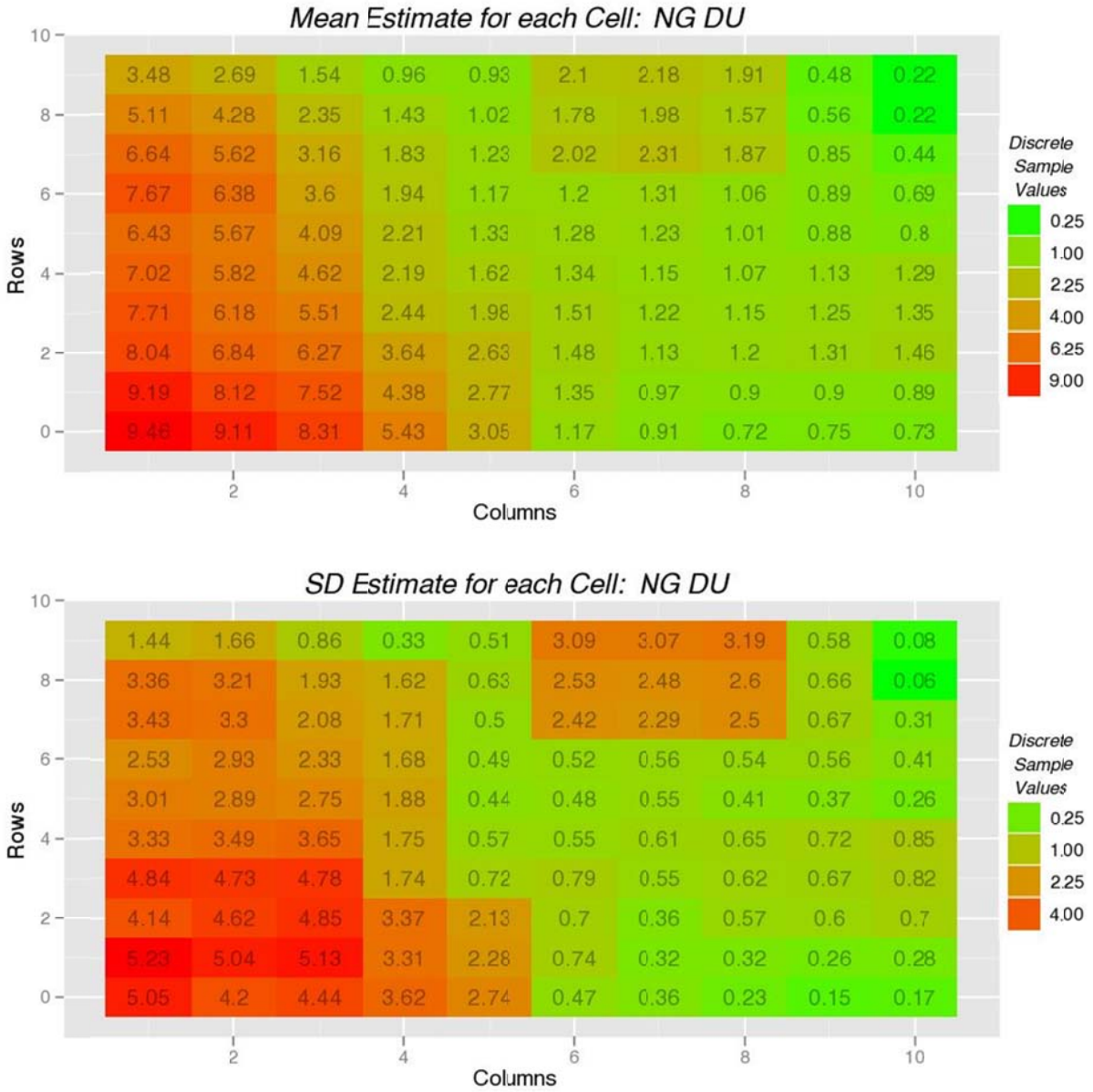


Figure A-9. Spatial mean and standard deviation estimates based on the discrete samples obtained during a study at CFB Petawawa, Canada (see Figure A-6). These values are used to identify the parameters for a unique Gamma distribution for each 1x1 m cell.

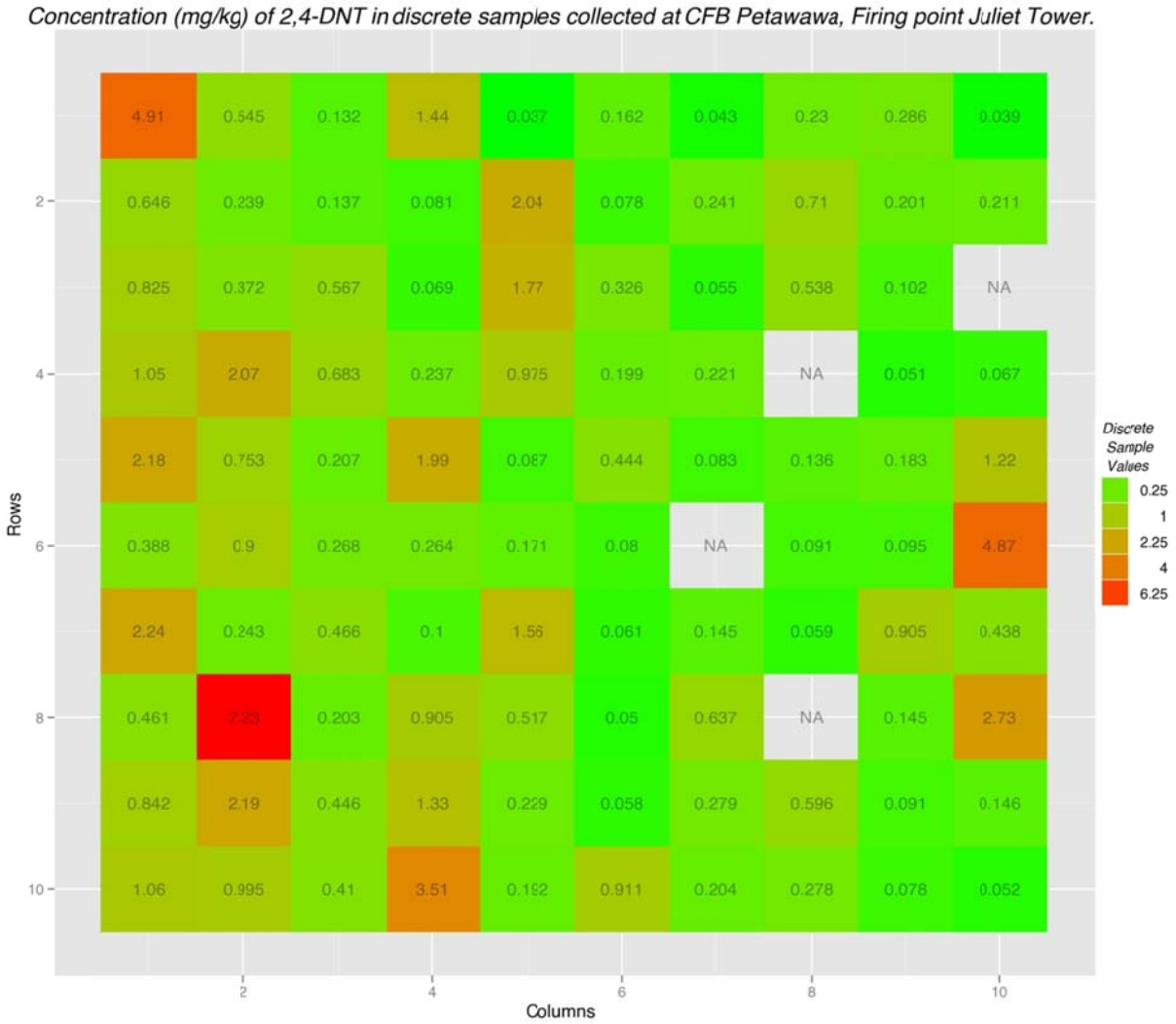
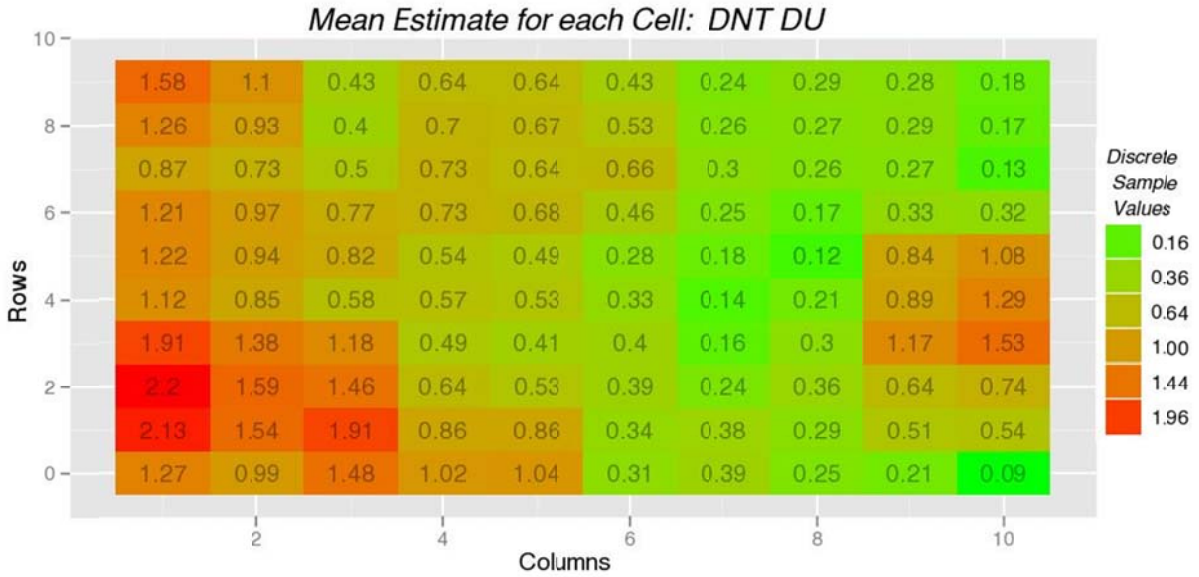


Figure A-10. Discrete data gathered from a Hand Grenade Range at CFB Petawawa, Canada within a 10 x 10 m decision unit were a discrete sample was collected from each node of a 10 x 10 grid. All 2-4 DNT values are in mg/kg.



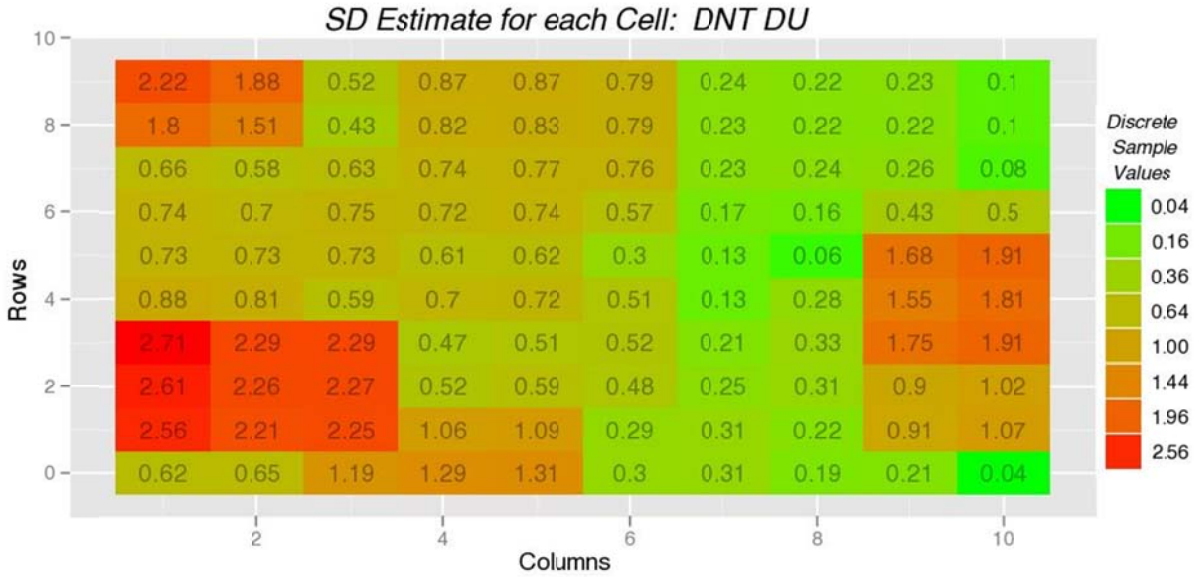


Figure A-11. Spatial mean and standard deviation estimates based on the discrete samples obtained during a study at CFB Petawawa, Canada (see Figure A-6). These values are used to identify the parameters for a unique Gamma distribution for each 1x1 m cell.

Appendix B – Simulation Results on other ITRC Developed Sites

This appendix includes results of the sampling methods described in this report applied to another sites developed by an ITRC team member. Anita Singh simulated multiple site distribution patterns. The work presented below is for one of her simulated sites. Anita Singh distributed discrete points on a map (see Figure B-1). The simulation routines developed for this report are based on sampling for sites with simulated increment values. Thus, we took her spatial discrete point map and combined points into what could be used as increments. We combined the points in two different methods as the appropriate units were not assigned to the simulated values. Each method took the simulated points within the site and identified all possible increments and which points fell into each increment that could be sampled from the site. With the points assigned to each increment the first combination was to average all points within the increment (See Figure B-2) and the second combination was to sum all points within an increment (See Figure B-7). Any increment location that did not contain points was assigned a value of zero to allow all spatial locations to have a value and an equal likelihood of being selected. The results are shown for the averaging method first and the sum method follows.

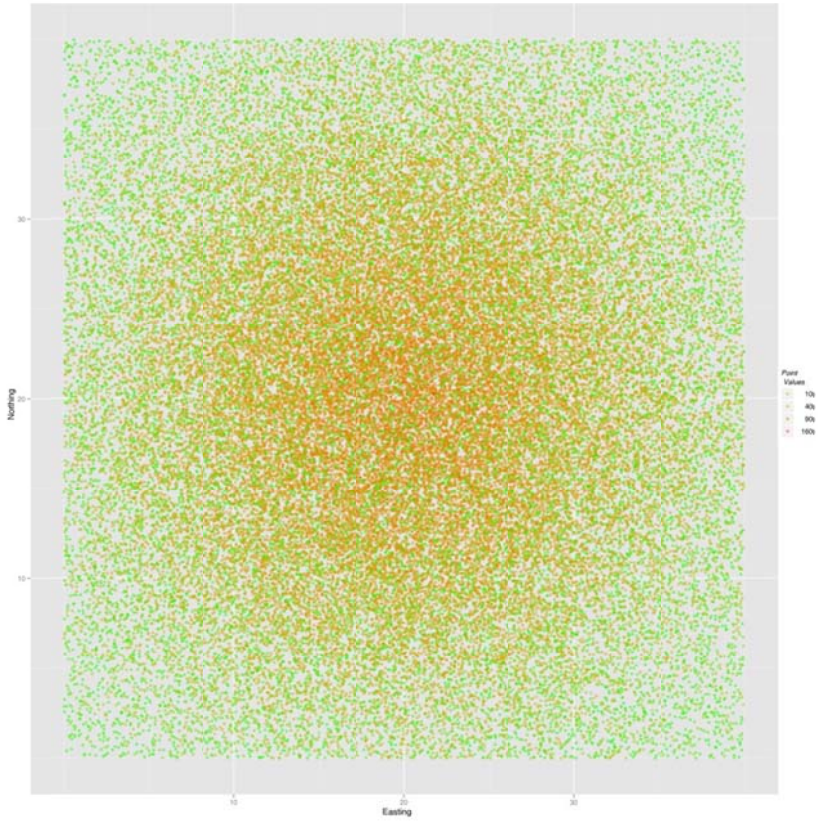


Figure B-1. Plot of Dr. Singh's raw point data dispersed within her decision unit.

Averaged Increment Results

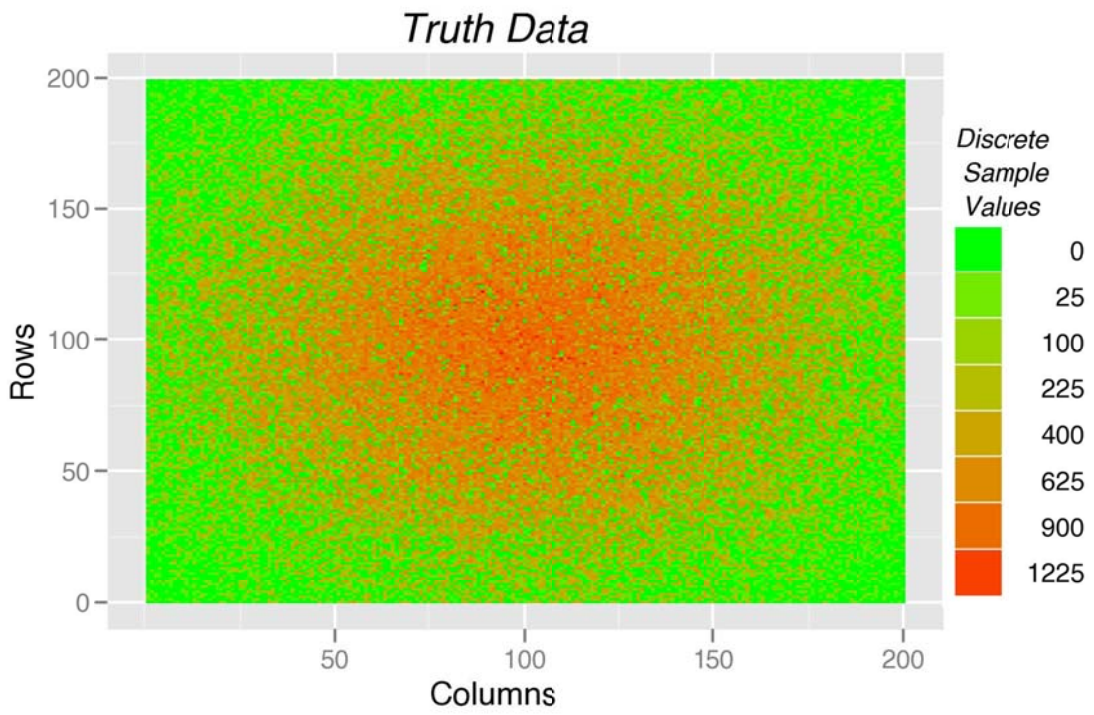


Figure B-2. Spatial map of averaged increment values using simulated discrete points.

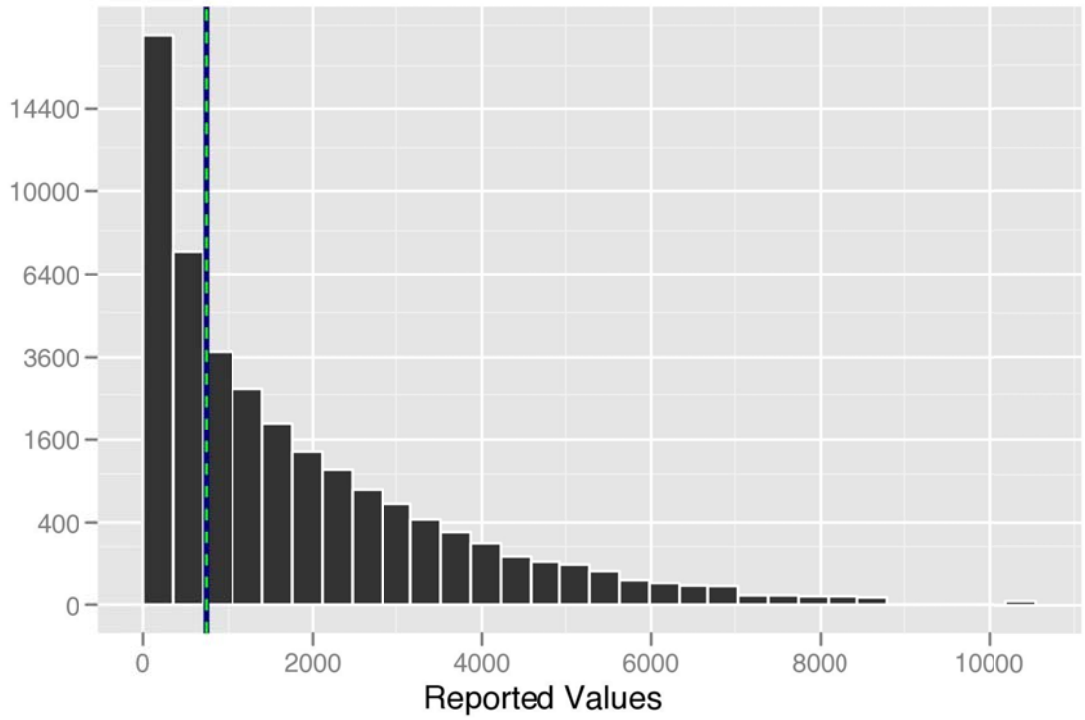


Figure B-3. Distributional plot of the averaged points for each discrete/increment within the DU.

Table B-1. Discrete summary: Anita Means

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	99.75	96.45	146.80	63.84	12.59	16.11
Systematic	9	99.95	98.35	149.38	63.42	51.35	16.47
Random	16	100.00	97.70	109.53	43.91		7.73
Systematic	16	99.95	98.40	110.71	44.38	5.56	8.16
Random	30	100.00	97.95	80.47	31.53		5.84
Systematic	30	100.00	98.30	80.44	31.32		5.05
Random	100	100.00	98.40	43.66	16.44		2.28
Systematic	100	100.00	98.65	43.83	16.57		4.28

Table B-2. Standard IS summary: Anita Means

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	93.40	95.65	46.28	63.35	7.66	8.25
3	16	98.20	96.00	40.18	27.05	4.72	5.35
4	16	98.80	94.60	35.72	19.78	3.70	3.89
5	16	99.20	95.40	32.60	16.32	3.94	4.10
2	30	94.40	95.90	35.15	48.25	5.92	5.90
3	30	97.60	95.45	29.72	20.09	3.15	3.75
4	30	99.10	95.15	26.48	14.90	3.09	2.69
5	30	99.50	96.00	24.43	12.52	3.29	3.23
2	49	95.80	97.25	28.85	39.05	4.20	4.67
3	49	98.90	97.50	25.77	17.98	2.18	2.31
4	49	99.35	97.80	22.80	13.35	1.85	2.27
5	49	99.70	97.95	21.03	11.33	1.07	2.08
2	100	94.35	95.95	19.33	26.68	3.14	3.42
3	100	97.85	95.55	16.93	11.40	2.33	2.34
4	100	98.75	95.30	14.80	8.16	2.40	2.18
5	100	99.45	96.95	13.30	6.52	1.21	1.88

Table B-3. Grouped IS summary: Anita Means

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	93.65	95.35	54.49	75.10	8.73	8.60
3	16	100.00	100.00	124.33	81.23		
4	16	98.75	94.05	47.05	26.49	3.67	5.58
5	16	100.00	100.00	103.60	49.39		
2	30	93.70	95.55	41.56	57.04	6.27	6.11
3	30	100.00	100.00	122.60	80.12		
4	30	99.10	95.45	35.95	19.97	5.11	4.35
5	30	100.00	100.00	101.61	48.45		
2	49	92.70	94.95	32.80	44.36	4.56	4.89
3	49	100.00	100.00	125.56	82.51		
4	49	99.00	95.30	28.10	15.53	4.07	3.69
5	49	100.00	100.00	103.42	49.97		
2	100	93.20	94.90	22.13	30.49	4.15	3.85
3	100	100.00	100.00	123.41	80.74		
4	100	98.75	94.65	19.03	10.56	2.02	2.23
5	100	100.00	100.00	99.59	47.49		

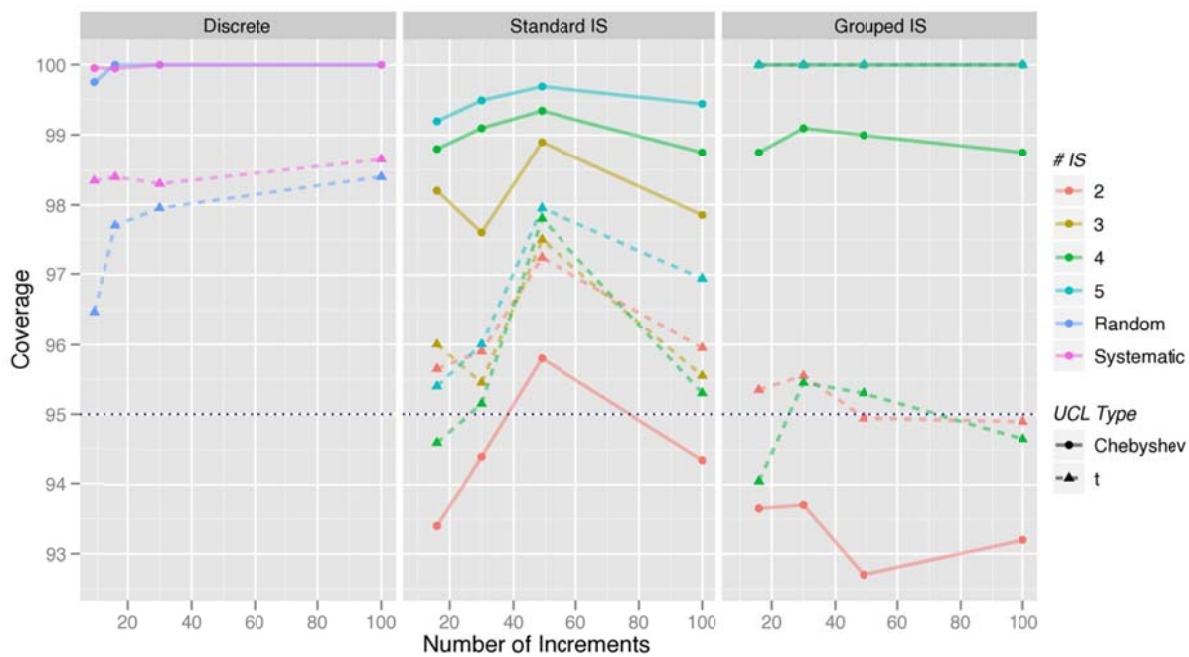


Figure B-4. Coverage plots of the different sampling patterns performance using a t-distribution and Chebyshev UCL multiplier.

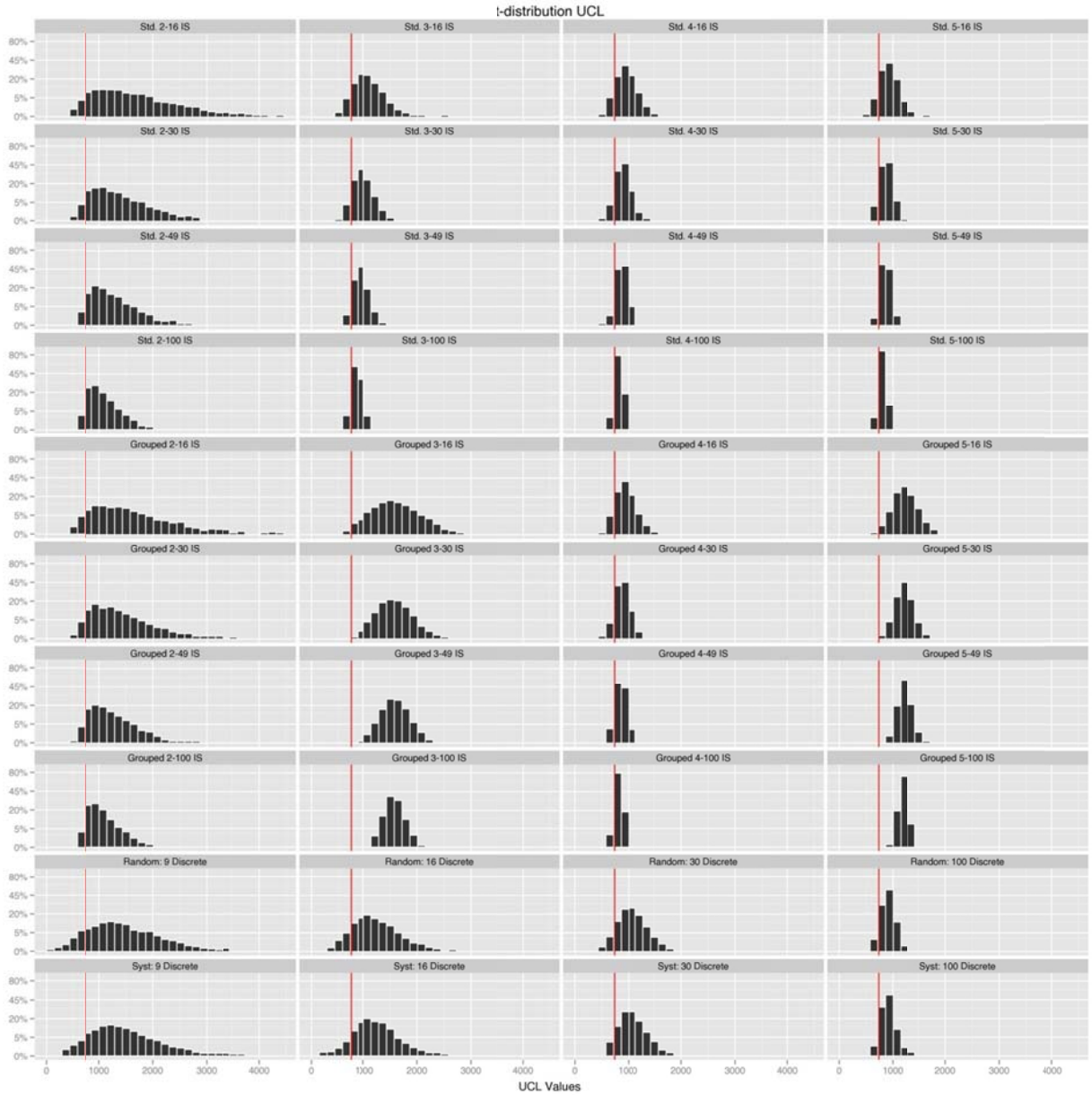


Figure B-5. Distribution of the t-distribution UCLs for each of the 40 different patterns.

Summed Increment Results

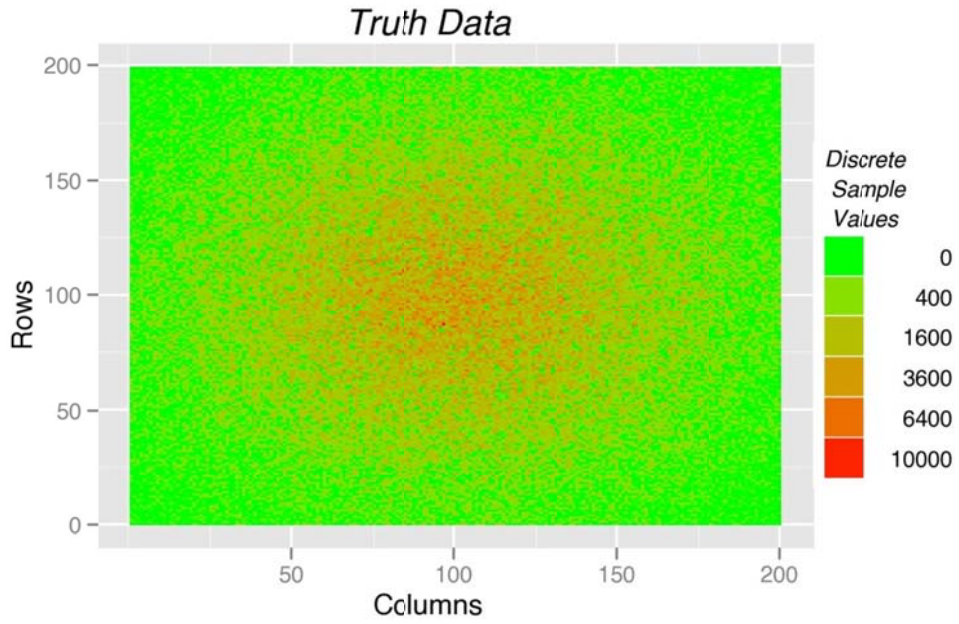


Figure B-6. Spatial map of the summed increment values.

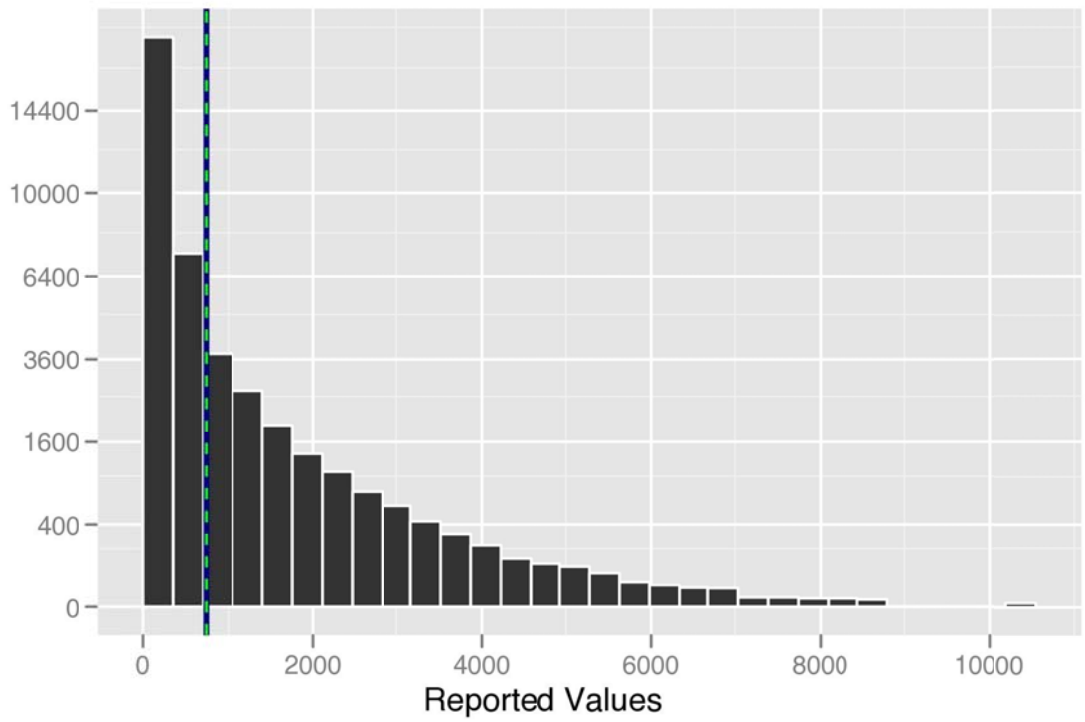


Figure B-7. Distributional histogram of the summed increment values.

Table B-4. Discrete summary: Anita Sums

Grid Sampling Type	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
Random	9	98.95	89.75	197.78	93.23	21.44	18.65
Systematic	9	99.20	93.60	200.89	90.44	11.47	18.19
Random	16	99.60	92.25	146.78	62.80	9.29	14.10
Systematic	16	99.45	95.45	153.19	64.01	19.43	17.37
Random	30	99.95	95.75	112.20	45.54	8.53	9.37
Systematic	30	100.00	96.05	111.55	44.84		7.59
Random	100	100.00	97.10	61.26	23.48		5.05
Systematic	100	100.00	97.55	60.40	22.69		5.35

Table B-5. Standard IS summary: Anita Sums

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	92.05	94.30	67.44	92.35	11.61	12.11
3	16	97.05	94.00	56.96	38.74	6.40	7.19
4	16	98.35	94.70	51.47	28.74	6.34	7.90
5	16	98.75	92.80	45.01	22.94	6.10	6.10
2	30	91.70	94.15	50.64	69.07	8.73	9.18
3	30	96.50	93.30	43.91	30.21	4.90	5.75
4	30	98.65	94.40	38.75	21.92	3.30	4.44
5	30	99.60	94.45	35.28	18.25	3.07	3.73
2	49	94.95	96.60	40.76	55.11	4.97	5.28
3	49	98.30	96.30	36.12	25.17	3.21	3.62
4	49	99.55	97.35	32.31	18.78	5.32	3.55
5	49	99.75	97.30	29.41	15.87	2.08	2.74
2	100	91.95	93.80	28.08	38.55	4.76	4.60
3	100	97.15	93.95	23.91	16.34	2.98	3.04
4	100	98.60	93.30	21.00	11.83	2.61	2.60
5	100	99.40	93.45	19.08	9.81	1.24	1.90

Table B-6. Grouped IS summary: Anita Sums

Number of IS	Number of Increments	Chebyshev UCL Coverage	t UCL Coverage	Chebyshev RPD Above Mean	t RPD Above Mean	Chebyshev RPD Below Mean	t RPD Below Mean
2	16	91.25	93.55	79.28	108.33	13.79	13.85
3	16	100.00	99.85	173.64	113.57		1.78
4	16	98.40	92.55	65.37	37.25	8.17	7.44
5	16	100.00	99.80	139.01	66.35		4.96
2	30	92.45	94.75	57.57	79.00	9.27	10.02
3	30	100.00	100.00	170.43	111.38		
4	30	98.10	92.60	48.86	27.52	5.54	6.38
5	30	100.00	100.00	135.80	64.72		
2	49	93.10	94.95	47.16	64.46	7.42	7.34
3	49	100.00	100.00	175.27	115.00		
4	49	98.30	93.80	38.27	21.31	3.35	4.91
5	49	100.00	100.00	138.20	66.60		
2	100	92.45	94.70	30.13	41.38	5.71	5.89
3	100	100.00	100.00	175.60	114.81		
4	100	98.90	94.35	25.72	14.28	3.67	2.85
5	100	100.00	100.00	132.74	63.22		

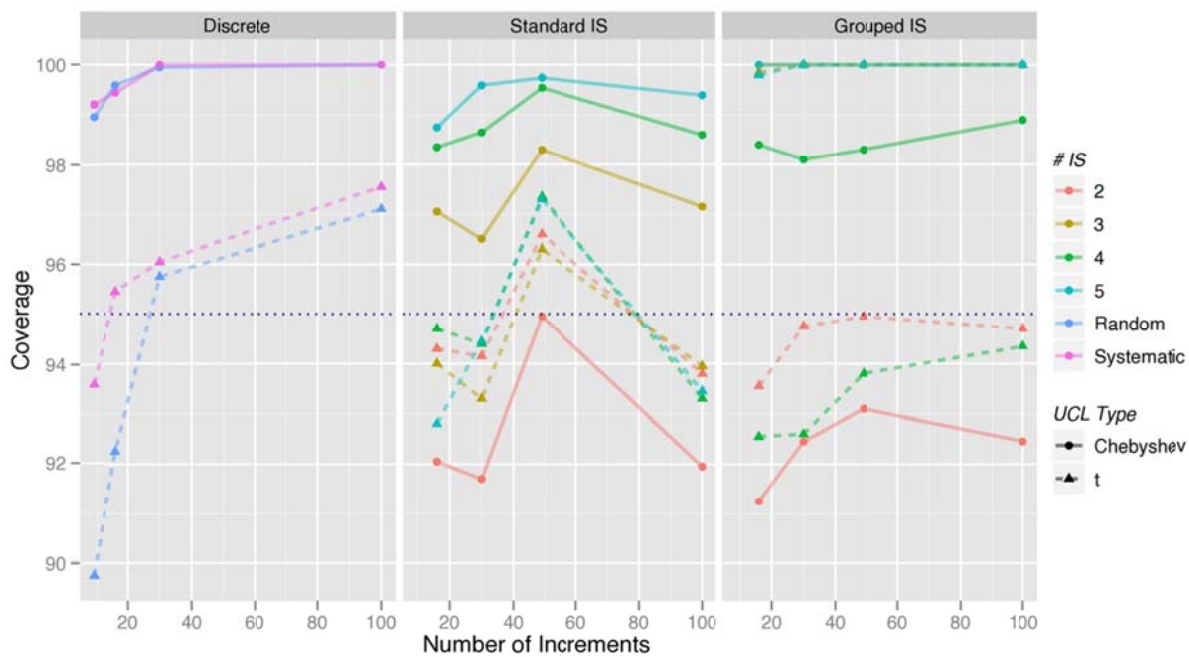


Figure B-8. Coverage plot of the performance of the different sampling patterns using a t-distribution and a Chebyshev UCL multiplier.

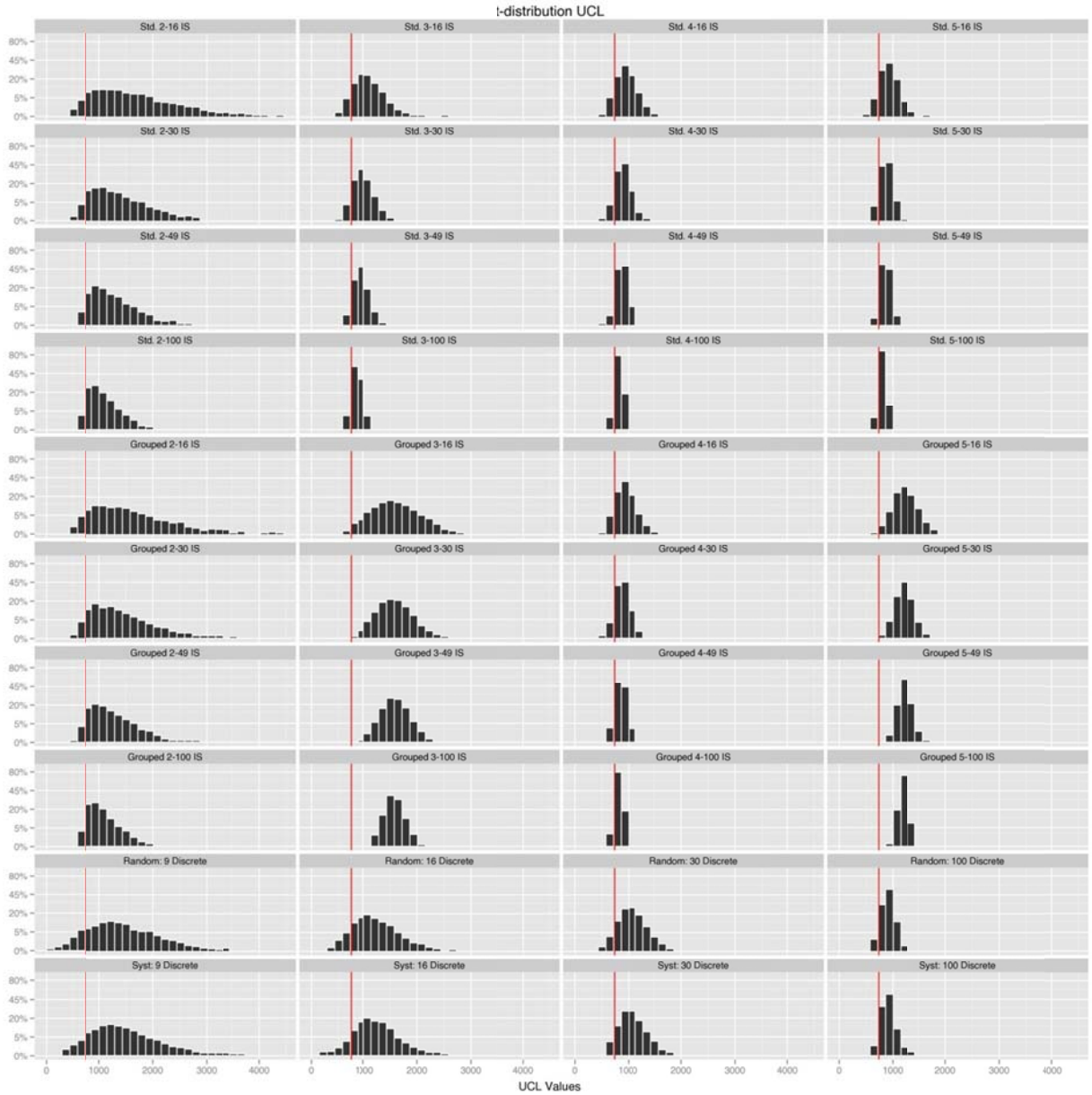


Figure B-9. Distribution of the t-distribution UCLs for each of the 40 different patterns.